Towards the Design of Better Equity Benchmarks

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The EDHEC Risk Institute is dedicated to the production and international diffusion of academic research relevant to the investment community, at a time when the industry is affected by a number of profound paradigm shifts and when academic guidance can be of some usefulness.

The goal of this particular presentation is to provide an overview of the latest results of our research program on “Indices and Benchmarks”.

Other research programs:
- ALM and Asset Management
- Asset Allocation and Alternative Diversification
- Asset Management and Derivatives Instruments
- Performance and Style Analysis
- Best Execution and Operational Performance
Problems with Existing Equity Indices

Rehabilitating the Tangency Portfolio

Implementation and Empirical Results

FTSE EDHEC-Risk Efficient Index Series
The standard indices weight stocks proportional to their market cap.

Cap-weighting is often believed to lead to risk/reward efficient portfolios.

This belief is based on the Capital Asset Pricing Model (CAPM).
- The CAPM assumes that each investor holds the same efficient tangency portfolio, and therefore concludes that the aggregate portfolio held by investors (which by definition is cap weighted) is also efficient.

The strong message from the CAPM: you do not need to gather any information on covariances and expected returns to find optimal portfolios (because everybody else does)!
Problems with Existing Indices

**Lack of Mean-Variance Efficiency**

- Being able to access efficient portfolios without gathering any information on risk and return may sound too good to be true!


  Haugen and Baker (1991):
  
  “Cap-weighted stock portfolios are inefficient investments.”
  “Even the most comprehensive cap-weighted portfolios occupy positions inside the efficient set.”

  Cochrane (2001):
  
  “Market indices [...] are if anything inside that [mean-variance] frontier”
Problems with Existing Indices

Inefficiency - Empirical Findings

- Cap-weighted index lies deep inside the ex-post efficient frontier.

Based on data for the period 1979-1998. The efficient frontier assumes a perfect forecast of the future covariance matrix and of the future mean return. Figure taken from Schwartz (2000), Figure 3, page 19.
Problems with Existing Indices

Cap Weighted versus Equally-Weighted Portfolios

Problems with Existing Indices  
**Inefficiency - Theoretical Reasons**

- It is not surprising that market cap weighted indices are not efficient portfolios.
  - When relaxing the highly unrealistic assumptions of the CAPM (such as identical preferences and time horizons, frictionless markets), financial theory does not predict that the market portfolio is efficient (Sharpe (1991), Markowitz (2005)).
  - Besides, even if the world worked according to the CAPM theory, standard equity indices do not represent the true market portfolio (which reflects all wealth in the economy as opposed to only the major stocks).
Problems with Existing Indices

Inefficiency - Theoretical Reasons

- Today, multi-factor models are preferred to the CAPM in both academia (e.g., Fama-French 3 factor model) and practice (e.g., Barra).

- But if there are factors other than the “market” return that yield a risk premium, the mean-variance optimal portfolio is no longer cap-weighted (Merton (1971), Cochrane (1999)).

  → How can index providers who furnish style or factor indices claim that their cap-weighted market indices are optimal?
  → Why are those style indices cap-weighted?

- From a “post-CAPM” view, cap-weighting is an ad-hoc weighting scheme.
Problems with Existing Indices

Inefficiency – Practical Reasons

- Cap-weighting is particularly inefficient because it leads to high concentration: the effective number of stocks in the index is low.

\[ \tilde{n} = \frac{1}{\sum_{i=1}^{n} w_i^2} . \]

The effective number of stocks is the reciprocal of the Herfindhal index, a measure of portfolio concentration.

<table>
<thead>
<tr>
<th>Index</th>
<th>Nominal number</th>
<th>Effective number</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P</td>
<td>500</td>
<td>94</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>100</td>
<td>37</td>
</tr>
<tr>
<td>FTSE 100 (UK)</td>
<td>100</td>
<td>28</td>
</tr>
<tr>
<td>FTSE Eurobloc</td>
<td>300</td>
<td>104</td>
</tr>
<tr>
<td>FTSE Japan</td>
<td>500</td>
<td>103</td>
</tr>
</tbody>
</table>

*Average effective number based on quarterly assessment for the time period 01/1959 to 12/2008 for the S&P, 01/1975 to 12/2008 for the NASDAQ, and 12/2002 to 12/2008 for the other indices.*
- Problems with Existing Equity Indices
- Rehabilitating the Tangency Portfolio
- Implementation and Empirical Results
- FTSE EDHEC-Risk Efficient Index Series
Market cap weighted indices may be OK as indices, but they are not good choices as benchmarks because they are not efficient portfolios.

For a rational investor, the goal is to have a benchmark that delivers the best reward per unit of risk: Maximum Sharpe Ratio portfolio or tangency portfolio.

In the end, if one cares for a high reward-to-risk ratio, one should aim at maximizing the reward-to-risk ratio, which requires:

- Estimates for risk parameters
- Estimates for expected return parameters
Rehabilitating the Tangency Portfolio

Designing Investable Proxies for MSR Portfolios

The true tangency portfolio is a function of the (unknown) true parameter values

\[ w_{MSR} = f(\mu_i, \sigma_i, \rho_{ij}) \]

Implementable proxies depend on estimated parameter values

\[ \hat{w}_{MSR} = f(\hat{\mu}_i, \hat{\sigma}_i, \hat{\rho}_{ij}) \]
Rehabilitating the Tangency Portfolio

A Note on “Estimating” Risk and Return Parameters

- To construct mean-variance efficient portfolios, one requires estimates for expected returns, volatilities and correlations.

- Estimation is not forecasting:
  - *Estimation*: from a sample, we find parameter values that describe the behaviour of and relation between returns of constituent stocks;
  - *Forecasting*: we project the path of the future.

- Portfolio optimisation is not about predicting the future, it is hoping that the future will structurally resemble the past.

- Robust parameter estimation is about using information about this structure.
Rehabilitating the Tangency Portfolio

Estimating Risk Parameters

- Sophisticated statistical techniques can be used to generate decent risk estimates.

- The key challenge is curse of dimensionality:
  - The proven answer for this problem are factor model estimators for the variance-covariance matrix.
  - One can however make various choices in terms of the factor model

- Additional techniques can provide marginal improvements (e.g. GARCH & Markov Regime Switching - MRS- models to capture time and state-dependencies).
Rehabilitating the Tangency Portfolio

Estimating Risk Parameters (continued)

- In principle, one can chose between two types of factor models:
  - Explicit factors
    - Advantage: intuitive grasp of factors
    - Drawback: Factor selection risk; the model will be misspecified since we do not know the true factors.
  - Implicit factors: “Let the data talk”:
    - Advantage: Avoids taking a view on which factors matter.
    - Drawback: One may recover factors that don’t matter (sample risk).

- Efficient Index approach:
  - PCA summarises the maximum amount of information contained in the data with a set of uncorrelated implicit factors (linear combinations of the stocks’ returns).
  - For robustness, we limit the number of factors using an optimal choice criterion.
While statistical techniques are useful for estimating risk, statistics do not allow us to estimate expected returns reliably.

Direct estimation of expected returns from past returns is close to useless (Merton 1980)

What can we do in practice?
- “Give up” on expected return estimation
- Estimate expected returns indirectly
Rehabilitating the Tangency Portfolio

*Alternative I to Using Past Returns: GMV*

- “Give up”: **Global Minimum Variance (GMV)**
  - “Avoids” expected return estimation to rely solely on covariance matrix.
  - 1\textsuperscript{st} order problem: for this to be optimal, one needs to assume that all expected returns are identical - Very unrealistic assumption!
  - 2\textsuperscript{nd} order problem: leads to overweighting of low volatility stocks.
    - Such stocks come with low returns and high extreme risks.
    - This leads to sector biases (e.g., towards utility stocks).
Rehabilitating the Tangency Portfolio

Alternative II to Using Past Returns: Risk-based Estimation

- Risk-based estimation: *Efficient Indexation*

  - Based on common sense: *risk-return tradeoff* implies that expected return parameters should be positively related to risk parameters.
  
  - Economic theory can help identify the relevant risk indicators:
    - Rewarded exposure to multiple *systematic* risk factors (APT);
    - *Specific* risk may be rewarded (Merton (1987), Malkiel and Xu (2002));
    - Higher moment risk is also rewarded (many references).
## Rehabilitating the Tangency Portfolio

**On the Relationship between Downside Risk & Expected Returns**

Evidence that stock downside risk is related to expected returns

<table>
<thead>
<tr>
<th>Authors</th>
<th>Risk Measure</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang (2005)</td>
<td>Skewness</td>
<td>+</td>
</tr>
<tr>
<td>Boyer, Mitton and Vorkink (2009)</td>
<td>Skewness</td>
<td>+</td>
</tr>
<tr>
<td>Tang and Shum (2003)</td>
<td>Skewness (but not kurtosis)</td>
<td>+</td>
</tr>
<tr>
<td>Conrad, Dittmar and Ghysels (2009)</td>
<td>Skewness (but not kurtosis)</td>
<td>+</td>
</tr>
<tr>
<td>Ang et al. (2006)</td>
<td>Downside correlation</td>
<td>+</td>
</tr>
<tr>
<td>Huang et al (2009)</td>
<td>Value-at-Risk (EVT)</td>
<td>+</td>
</tr>
<tr>
<td>Bali and Cakici (2004)</td>
<td>Value-at-Risk (Historical)</td>
<td>+</td>
</tr>
<tr>
<td>Chen et al. (2009)</td>
<td>Semi-deviation</td>
<td>+</td>
</tr>
<tr>
<td>Estrada (2000)</td>
<td>Semi-deviation</td>
<td>+</td>
</tr>
</tbody>
</table>
Rehabilitating the Tangency Portfolio

*Expected Return Estimation in Efficient Indexation*

- **Pragmatic**
  - Relaxes a strong *implicit* assumption behind the GMV (that all expected returns are equal) and acknowledges that expected returns differ across stocks.

- **Risk-based**
  - No direct expected return estimation.
  - Differences in expected returns are assessed by looking at stocks’ riskiness, more specifically downside risk.
  - The *explicit* working assumption is that risk is rewarded.

- **Parsimonious and robust**
  - Expected returns are not assigned stock by stock but only to groups of stocks (deciles).
  - The expected return proxy is updated only infrequently (long horizon perspective to avoid “noise”).
Ten portfolios containing an equal number of stocks traded in the US (NYSE, NASDAQ and Alternext) are built every month after sorting the stocks based on their semi-deviation (calculated using daily data for last 30 months); the cumulative returns of each of these portfolios are calculated for various holding periods and averaged across the portfolio formation date. We use an event study methodology.
- Problems with Existing Equity Indices
- Rehabilitating the Tangency Portfolio
- Implementation and Empirical Results
- FTSE EDHEC-Risk Efficient Index Series
Empirical Tests

Methodology

- We go back to the basics of Modern Portfolio Theory: A formal Sharpe ratio maximisation allows generating a proxy for the tangency portfolio

- We use the robust input parameters described above:
  - Indirect expected return estimate based on median downside risk
  - Covariance matrix estimate based on an implicit factor model (PCA)

- We incorporate practical constraints:
  - Accounting for the presence of robustness and liquidity constraints through the introduction of min and max weights;
  - Accounting for the presence of turnover constraints through optimal control techniques (Leland 1999, Martellini and Priaulet 2002). Objective is a 30% maximum annual one way turnover.

- Our back test is based on long-term US data (out-of-sample performance from January 1959).
### Empirical Tests

#### Long-Term US Results

<table>
<thead>
<tr>
<th>Index</th>
<th>Ann. average return</th>
<th>Ann. std. Deviation</th>
<th>Sharpe Ratio</th>
<th>Information Ratio</th>
<th>Tracking Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficient Index</td>
<td>11.63%</td>
<td>14.65%</td>
<td>0.41</td>
<td>0.52</td>
<td>4.65%</td>
</tr>
<tr>
<td>Cap-weighted</td>
<td>9.23%</td>
<td>15.20%</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>Difference (Efficient minus Cap-weighted)</td>
<td>2.40%</td>
<td>-0.55%</td>
<td>0.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>p-value for difference</td>
<td>0.14%</td>
<td>6.04%</td>
<td>0.04%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The table shows risk and return statistics portfolios constructed with using the same set of constituents as the cap-weighted S&P 500 index. Rebalancing is quarterly subject to an optimal control of portfolio turnover (by setting the reoptimisation threshold to 50%). Portfolios are constructed by maximising the Sharpe ratio given an expected return estimate and a covariance estimate. The expected return estimate is set to the median total risk of stocks in the same decile when sorting on total risk. The covariance matrix is estimated using an implicit factor model for stock returns. Weight constraints are set so that each stock’s weight is between 1/2N and 2/N, where N is the number of index constituents. P-values for differences are computed using the paired t-test for the average, the F-test for volatility, and a Jobson-Korkie test for the Sharpe ratio. The results are based on weekly return data from 01/1959. We use a calibration period of 2 years and rebalance the portfolio every three months (at the beginning of January, April, July and October).
Empirical Tests

Results – Turnover and Concentration

<table>
<thead>
<tr>
<th>Index</th>
<th>Annual one-way turnover</th>
<th>Excess turnover vs. Cap-weighted</th>
<th>Average Effective constituents</th>
<th>Effective constituents to nominal constituents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficient Index</td>
<td>23.10%</td>
<td>18.41%</td>
<td>382</td>
<td>76%</td>
</tr>
<tr>
<td>Cap-weighted</td>
<td>4.69%</td>
<td>0.00%</td>
<td>94</td>
<td>19%</td>
</tr>
</tbody>
</table>

The table shows the resulting turnover measures for Efficient Indexation portfolios that have been implemented using the controlled reoptimisation with a threshold value of 50%. The table indicates the effective number of constituents in the efficient index and in the cap-weighted index, computed as the inverse of the sum of squared constituent weights. This measure is computed at the start of each quarter and averaged over the entire period. The results are based on weekly return data from 01/1959 to 12/2008.
Empirical Tests

Results – Evolution of Wealth Relative to CW

5 Year rolling performance of Cap-weighted and Efficient Index
Cumulative performance of Efficient Index and Cap-weighted index built from S&P500 constituents. 5 years of past weekly return data is used to calculate the cumulate returns every week. Analysis period is from 01/ 1959 to 12/2008.

The Efficient Index generally does not suffer from under-performance vs. cap-weighting over a five year period.
Empirical Tests

Phases of the business cycle and volatility regimes

<table>
<thead>
<tr>
<th></th>
<th>Ann. average return</th>
<th>Ann. volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Business cycle&quot;</td>
<td>Cap-weighting</td>
<td>Efficient indexation</td>
</tr>
<tr>
<td>Recessions</td>
<td>-1.64%</td>
<td>2.26%</td>
</tr>
<tr>
<td>Expansions</td>
<td>11.19%</td>
<td>13.30%</td>
</tr>
</tbody>
</table>

The table shows risk and return statistics computed for two sub-samples. The sub-samples are obtained by sorting the weekly observations based on a recession indicator for that week. The recession indicator is obtained from NBER dates for peaks and troughs of the business cycle. The results are based on weekly return data from 01/1959 to 12/2008.

<table>
<thead>
<tr>
<th></th>
<th>Ann. average return</th>
<th>Ann. volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Implied volatility regime&quot;</td>
<td>Cap-weighting</td>
<td>Efficient indexation</td>
</tr>
<tr>
<td>High volatility</td>
<td>8.90%</td>
<td>10.99%</td>
</tr>
<tr>
<td>Low volatility</td>
<td>6.22%</td>
<td>10.03%</td>
</tr>
</tbody>
</table>

The table shows risk and return statistics computed for two sub-samples of equal size. The sub-samples are obtained by sorting the weekly observations based on the value of the corresponding implied volatility index for that week. The median level of volatility is used to separate the two samples. The data for implied volatility indices start on 03/01/1986 (VXO index) and end on 26/12/2008.

Efficient indexation has higher average returns and lower volatility

- in expansions and recessions
- in high and low volatility regimes.
- Problems with Existing Equity Indices
- Rehabilitating the Tangency Portfolio
- Implementation and Empirical Results
- FTSE EDHEC-Risk Efficient Index Series
We have now moved from the R&D stage to the production stage through a partnership with FTSE.

This has led to the design of the FTSE EDHEC-Risk Efficient Index series:

- FTSE EDHEC-Risk Efficient USA Index
- FTSE EDHEC-Risk Efficient UK Index
- FTSE EDHEC-Risk Efficient Eurobloc Index
- FTSE EDHEC-Risk Efficient Developed Europe Index (also as “ex UK”)
- FTSE EDHEC-Risk Efficient Japan Index
- FTSE EDHEC-Risk Efficient Developed Asia Pacific ex Japan Index
- FTSE EDHEC-Risk Efficient Asia Pacific (also as “ex Japan”)
- FTSE EDHEC-Risk Efficient All World Index (also as “ex US” and “ex UK”)
- FTSE EDHEC-Risk Efficient Developed Index
- FTSE EDHEC-Risk Efficient Emerging Index
The FTSE EDHEC-Risk Efficient Indices are designed according to a methodology that is similar to the one in the long-term back test presented here, with a set of rules, validated by FTSE, that are adapted to the context of the production and live maintenance of an equity index.

The FTSE EDHEC-Risk Efficient Indices are based on all constituent securities in the FTSE All-World Index Series so that no selection bias is introduced.

The FTSE EDHEC-Risk Efficient Indices are reviewed quarterly based on the constituents of the underlying FTSE All-World Index available after the close of business on the third Friday of March, June, September and December.
FTSE EDHEC-Risk Efficient Index Series

Historical Performance

<table>
<thead>
<tr>
<th>Region</th>
<th>Ann. average return</th>
<th>Ann. std. dev.</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Efficient Index</td>
<td>Value Weighted</td>
<td>Diff.</td>
</tr>
<tr>
<td>USA</td>
<td>9.05%</td>
<td>5.59%</td>
<td>3.46%</td>
</tr>
<tr>
<td>Eurobloc</td>
<td>10.55%</td>
<td>7.22%</td>
<td>3.33%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>13.37%</td>
<td>8.99%</td>
<td>4.38%</td>
</tr>
<tr>
<td>Dev Asia Ex Japan</td>
<td>20.12%</td>
<td>18.96%</td>
<td>1.16%</td>
</tr>
<tr>
<td>Japan</td>
<td>5.17%</td>
<td>2.70%</td>
<td>2.46%</td>
</tr>
</tbody>
</table>

The table shows risk and return statistics computed for efficient indexation and cap-weighting applied to stock market index constituents in five regions. The statistics are based on weekly returns data from 23/12/2002 to 31/12/2009.

- The foundation paper, the official ground rules as well as other relevant information and related documents can be found at www.efficientindex.com
**FTSE EDHEC-Risk Efficient Index Series**

*Recent Performance: Past 5 Years*

<table>
<thead>
<tr>
<th>5Y Annual Performance</th>
<th>Efficient</th>
<th>Cap-weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Return</td>
<td>Volatility</td>
</tr>
<tr>
<td>Eurobloc</td>
<td>1.64%</td>
<td>22.71%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>8.82%</td>
<td>22.82%</td>
</tr>
<tr>
<td>United States</td>
<td>3.11%</td>
<td>23.90%</td>
</tr>
<tr>
<td>Japan</td>
<td>-6.65%</td>
<td>21.08%</td>
</tr>
<tr>
<td>Dev.Asia ex.Japan</td>
<td>12.98%</td>
<td>25.29%</td>
</tr>
</tbody>
</table>

As of September 30th 2010. Sharpe ratios are annualised assuming. Note that since negative Sharpe ratios cannot be interpreted, the Sharpe ratios are adjusted in the event that they are negative by replacing the risk-free rate with the average return on the cap-weighted index.

- Performance updates can be found at [www.efficientindex.com](http://www.efficientindex.com)
**FTSE EDHEC-Risk Efficient Index Series**

*Recent Performance: Year-to-Date*

<table>
<thead>
<tr>
<th>YTD Performance</th>
<th>Return (YTD)</th>
<th>Sharpe Ratio (YTD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Efficient</td>
<td>Cap-w.</td>
</tr>
<tr>
<td>Eurobloc</td>
<td>2.07%</td>
<td>-0.83%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>8.30%</td>
<td>5.08%</td>
</tr>
<tr>
<td>United States</td>
<td>8.46%</td>
<td>3.91%</td>
</tr>
<tr>
<td>Japan</td>
<td>-4.46%</td>
<td>-7.64%</td>
</tr>
<tr>
<td>Dev. Asia ex. Japan</td>
<td>16.18%</td>
<td>10.37%</td>
</tr>
</tbody>
</table>

As of September 30th 2010. Sharpe ratios are annualised assuming. Note that since negative Sharpe ratios cannot be interpreted, the Sharpe ratios are adjusted in the event that they are negative by replacing the risk-free rate with the average return on the cap-weighted index.

- Performance updates can be found at [www.efficientindex.com](http://www.efficientindex.com)
Conclusion

Improved Equity Benchmarks

- Cap-weighted indices are not efficient or well-diversified portfolios because they were never meant to be.

- Efficient Indices are one solution among others for designing better equity benchmarks.

- Originality of this solution: The Efficient Index explicitly aims at improving the Sharpe ratio.

“If one does not know to which port one is sailing, no wind is favorable.” (Seneca, circa 30 AD)
References

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