Towards the Design of Better Equity Benchmarks

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The EDHEC Risk Institute is dedicated to the production and international diffusion of academic research relevant to the investment community, at a time when the industry is affected by a number of profound paradigm shifts and when academic guidance can be of some usefulness.

The goal of this particular presentation is to provide an overview of the latest results of our research program on “Indices and Benchmarks”.

Other research programs: ALM and Asset Management; Asset Allocation and Alternative Diversification; Asset Management and Derivatives Instruments; Performance and Style Analysis; Best Execution and Operational Performance.
Problems with Existing Equity Indices

Rehabilitating the Tangency Portfolio

Implementation and Empirical Results

FTSE EDHEC-Risk Efficient Index Series
The standard practice of using stock market indices based on market cap weighting schemes as investment benchmarks has recently faced renewed criticism.

More than 15 years ago, a number of papers (e.g., Haugen and Baker (1991) and Grinold (1992)) have already offered empirical evidence that market-cap weighted indices provide an inefficient risk-return trade-off.

“Cap-weighted stock portfolios are inefficient investments. […] Even the most comprehensive cap-weighted portfolios occupy positions inside the efficient set.” (Haugen and Baker (1991))

“Market indices […] are if anything inside that [mean-variance] frontier”

(John Cochrane (2001))
Problems with Existing Indices

Inefficiency - Empirical Arguments

- Cap-weighted index lies deep inside the ex-post efficient frontier.

Based on data for the period 1979-1998. The efficient frontier assumes a perfect forecast of the future covariance matrix and of the future mean return. Figure taken from Schwartz (2000), Figure 3, page 19.
Problems with Existing Indices

Cap Weighted versus Equally-Weighted Portfolios

- Cap-weighted index
- Equally-weighted index

Expected Return vs. Volatility

True Tangency Portfolio
The belief in the efficiency of market cap weighted indices is based on some misperception about the Capital Asset Pricing Model (CAPM).

- CAPM assumes that each investor holds the same efficient tangency portfolio, and therefore concludes that the aggregate portfolio held by investors (which by definition is cap weighted) is also efficient.
- CAPM is a great piece of economic theory but CAPM assumptions (homogenous preferences & expectations, absence of frictions & non-tradable assets) and CAPM predictions (differences in betas explain differences in expected returns) can not be taken seriously.

Sharpe (1991) and Markowitz (2005) state that under real-world conditions the market portfolio may not be efficient.

Beside, even if the CAPM was the true asset pricing model, a given equity index is not a good proxy for the true market portfolio.
Problems with Existing Indices

Concentration - Effective Number of Stocks

- Burden of proof is reversed:
  - No good reason why cap-weighted stock index should be efficient;
  - Beside, it may be particularly inefficient because leads to high concentration.

<table>
<thead>
<tr>
<th>Index</th>
<th>Nominal number</th>
<th>Effective number</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P</td>
<td>500</td>
<td>94</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>100</td>
<td>37</td>
</tr>
<tr>
<td>Eurostoxx</td>
<td>300</td>
<td>98</td>
</tr>
<tr>
<td>Topix</td>
<td>500</td>
<td>94</td>
</tr>
</tbody>
</table>

Average effective number based on quarterly assessment for the time period 01/1959 to 12/2008 for the S&P, 01/1975 to 12/2008 for the NASDAQ, and 12/2002 to 12/2008 for the other indices.
Problems with Existing Equity Indices

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FTSE EDHEC-Risk Efficient Index Series
Rehabilitating the Tangency Portfolio

*Back to the Basics of Portfolio Theory*

- Market cap weighted indices may be OK as indices, but they are not good choices as benchmarks because they are not efficient portfolios.

- For a rational investor, the goal is to have a benchmark with the best risk-adjusted performance.

- In the end, if one cares for a high reward-to-risk ratio, one should aim at maximizing the reward-to-risk ratio, which requires:
  - Estimates for risk parameters;
  - Estimates for expected return parameters.
Rehabilitating the Tangency Portfolio

*Designing Investable Proxies for MSR Portfolios*

The true tangency portfolio is a function of the (unknown) **true** parameter values:

\[ w_{MSR} = f(\mu_i, \sigma_i, \rho_{ij}) \]

Implementable proxies depend on **estimated** parameter values:

\[ \hat{w}_{MSR} = f(\hat{\mu}_i, \hat{\sigma}_i, \hat{\rho}_{ij}) \]
Suitably designed statistical techniques have been found useful to generate decent risk estimates.

On the other hand, statistics is close to useless in terms of expected return estimation (Merton (1980)).

Common sense: *risk-return tradeoff* implies that expected return should be positively related to risk.

Economic analysis can help identify the relevant risk indicator:
- Linear relationship between beta & expected return (CAPM);
- Linear pricing relationship involving other factors (APT);
- Specific risk may also be rewarded (Merton (1987)) (*)
- Higher moment risk is also rewarded (many references).

**Rehabilitating the Tangency Portfolio**

*On the Relationship between Downside Risk & Expected Returns*

Evidence that stock downside risk is related to expected returns:

<table>
<thead>
<tr>
<th>Authors</th>
<th>Risk Measure</th>
<th>Relation</th>
<th>Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang (2005)</td>
<td>Skewness</td>
<td>+</td>
<td>Skew</td>
</tr>
<tr>
<td>Zhang (2005)</td>
<td>Skewness</td>
<td>+</td>
<td>Skew</td>
</tr>
<tr>
<td>Boyer, Mitton and Vorkink (2009)</td>
<td>Skewness</td>
<td>+</td>
<td>Skew</td>
</tr>
<tr>
<td>Tang and Shum (2003)</td>
<td>Skewness (but not kurtosis)</td>
<td>+</td>
<td>Skew</td>
</tr>
<tr>
<td>Connrad, Dittmar and Ghysels (2009)</td>
<td>Skewness (but not kurtosis)</td>
<td>+</td>
<td>Skew</td>
</tr>
<tr>
<td>Ang et al. (2006)</td>
<td>Downside correlation</td>
<td>+</td>
<td>Vol, Skew, Kurt</td>
</tr>
<tr>
<td>Chen et al. (2009)</td>
<td>Semi-deviation</td>
<td>+</td>
<td>Vol, Skew</td>
</tr>
</tbody>
</table>
- Problems with Existing Equity Indices
- Rehabilitating the Tangency Portfolio
- Implementation and Empirical Results
- FTSE EDHEC-Risk Efficient Index Series
Our objective is to go back to the basics of Modern Portfolio Theory to generate a proxy for the tangency portfolio.

Such a portfolio may provide investors with a more efficient way of extracting the equity risk premium from the stock market.

We perform a formal maximum Sharpe ratio portfolio optimization using suitable estimates for expected return and risk parameters (more details on this later).

Out back test is based on long-term US data (out-of-sample performance from January 1959).
The table shows risk and return statistics portfolios constructed with using the same set of constituents as the cap-weighted S&P 500 index. Rebalancing is quarterly subject to an optimal control of portfolio turnover (by setting the reoptimisation threshold to 50%). Portfolios are constructed by maximising the Sharpe ratio given an expected return estimate and a covariance estimate. The expected return estimate is set to the median total risk of stocks in the same decile when sorting on total risk. The covariance matrix is estimated using an implicit factor model for stock returns. Weight constraints are set so that each stock's weight is between $1/2N$ and $2/N$, where $N$ is the number of index constituents. P-values for differences are computed using the paired t-test for the average, the F-test for volatility, and a Jobson-Korkie test for the Sharpe ratio. The results are based on weekly return data from 01/1959. We use a calibration period of 2 years and rebalance the portfolio every three months (at the beginning of January, April, July and October).

<table>
<thead>
<tr>
<th>Index</th>
<th>Ann. average return</th>
<th>Ann. std. Deviation</th>
<th>Sharpe Ratio</th>
<th>Information Ratio</th>
<th>Tracking Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficient Index</td>
<td>11.63%</td>
<td>14.65%</td>
<td>0.41</td>
<td>0.52</td>
<td>4.65%</td>
</tr>
<tr>
<td>Cap-weighted</td>
<td>9.23%</td>
<td>15.20%</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>Difference (Efficient minus Cap-weighted)</td>
<td>2.40%</td>
<td>-0.55%</td>
<td>0.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>p-value for difference</td>
<td>0.14%</td>
<td>6.04%</td>
<td>0.04%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
## Empirical Tests

### Results – Turnover and Concentration

The table shows the resulting turnover measures for Efficient Indexation portfolios that have been implemented using the controlled reoptimisation with a threshold value of 50%. The table indicates the effective number of constituents in the efficient index and in the cap-weighted index, computed as the inverse of the sum of squared constituent weights. This measure is computed at the start of each quarter and averaged over the entire period. The results are based on weekly return data from 01/1959 to 12/2008.

<table>
<thead>
<tr>
<th>Index</th>
<th>Annual one-way turnover</th>
<th>Excess turnover vs. Cap-weighted</th>
<th>Average Effective constituents</th>
<th>Effective constituents to nominal constituents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficient Index</td>
<td>23.10%</td>
<td>18.41%</td>
<td>382</td>
<td>76%</td>
</tr>
<tr>
<td>Cap-weighted</td>
<td>4.69%</td>
<td>0.00%</td>
<td>94</td>
<td>19%</td>
</tr>
</tbody>
</table>

The table shows the resulting turnover measures for Efficient Indexation portfolios that have been implemented using the controlled reoptimisation with a threshold value of 50%. The table indicates the effective number of constituents in the efficient index and in the cap-weighted index, computed as the inverse of the sum of squared constituent weights. This measure is computed at the start of each quarter and averaged over the entire period. The results are based on weekly return data from 01/1959 to 12/2008.
Empirical Tests

Results – Evolution of Wealth

- Prolonged lower returns occurred in the bull market of the late 1990s.

- This underperformance happened as the cap-weighted index returned in excess of 20% annual.

- Even in this period, efficient indexation had lower volatility than cap-weighting.
- Problems with Existing Equity Indices
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- FTSE EDHEC-Risk Efficient Index Series
We have now moved from R&D stage to production stage through a partnership with FTSE.

This has led to the design of the FTSE EDHEC-Risk Efficient Index series:
- FTSE EDHEC-Risk Efficient UK Index
- FTSE EDHEC-Risk Efficient Eurobloc Index
- FTSE EDHEC-Risk Efficient Developed Asia Pacific ex Japan Index
- FTSE EDHEC-Risk Efficient Japan Index
- FTSE EDHEC-Risk Efficient USA Index
The FTSE EDHEC-Risk Efficient Indices are designed according to a methodology that is similar to the one in the long-term back test presented here, with a set of rules, validated by FTSE, that are adapted to the context of the production and live maintenance of an equity index.

The FTSE EDHEC-Risk Efficient Indices are based on all constituent securities in the FTSE All-World Index Series so that no selection bias is introduced.

The FTSE EDHEC-Risk Efficient Indices are reviewed quarterly based on the constituents of the underlying FTSE All-World Index available after the close of business on the third Friday of March, June, September and December.
In terms of covariance matrix estimate, we use an implicit factor model. (*)

In terms of expected return estimates, stocks will be grouped into portfolios and we use the median downside risk estimate (semi-deviation) of stocks in the portfolio as an estimate for the expected return for each stock in this portfolio.

We additionally incorporate the following ingredients:

- Accounting for the presence of robustness and liquidity constraints through the introduction of min and max weights;
- Accounting for the presence of turnover constraints through optimal control techniques (**) (30% max annual one way turnover);

(*) We use random matrix theory for obtaining the optimal number of factors.
(**) See Leland (1999), or Martellini and Priaulet (2002).
The table shows risk and return statistics computed for efficient indexation and cap-weighting applied to stock market index constituents in five regions. The statistics are based on weekly returns data from 23/12/2002 to 31/12/2009.

- The foundation paper, the official ground rules as well as other relevant information and related documents can be found at [http://www.edhec-risk.com/indexes/efficient](http://www.edhec-risk.com/indexes/efficient).

<table>
<thead>
<tr>
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<th>Ann. average return</th>
<th>Ann. std. dev.</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Efficient Index</td>
<td>Value Weighted</td>
<td>Diff.</td>
</tr>
<tr>
<td>USA</td>
<td>9.05%</td>
<td>5.59%</td>
<td>3.46%</td>
</tr>
<tr>
<td>Eurobloc</td>
<td>10.55%</td>
<td>7.22%</td>
<td>3.33%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>13.37%</td>
<td>8.99%</td>
<td>4.38%</td>
</tr>
<tr>
<td>Dev Asia Ex</td>
<td>20.12%</td>
<td>18.96%</td>
<td>1.16%</td>
</tr>
<tr>
<td>Japan</td>
<td>5.17%</td>
<td>2.70%</td>
<td>2.46%</td>
</tr>
</tbody>
</table>
Conclusion

- Cap-weighted indices are not **efficient** or **well-diversified** portfolios because they were never meant to be; the main objective of these indices is to **represent** the stock market, thus neglecting the need for the most efficient risk-return trade-off.

- Alternative weighting schemes do not explicitly aim at improving the risk-reward ratio either.

- The efficient index series uses robust estimates of expected returns and covariance as inputs in a maximisation of the reward-to-risk ratio.

- Out-of-sample reward-to-risk ratios are higher than for the value-weighted index.

- Performance is consistent across different time periods and geographical zones.
References

- Barberis, N. and M. Huang, Stocks as lotteries: The implications of probability weighting for security prices, 2007, working paper.
Schwartz, T., 2000, How to Beat the S&P500 with Portfolio Optimization, DePaul University, working paper.
Zhang, Y., 2005, Individual Skewness and the Cross-Section of Average Stock Returns, Yale University, working paper.
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