Academia meets Practice

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Asset Allocation Decisions in the Presence of Regime Switches

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Main questions

- Are there regimes in financial asset returns?
- How can we detect and estimate these regimes?
- Are regimes important for asset allocation decisions?

Outline

- Regime-switching models and financial markets
- A simple volatility model with two regimes
- Regime switches and international diversification
- A classical allocation model between stocks, bonds and Treasury bills

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Stylized facts on financial markets

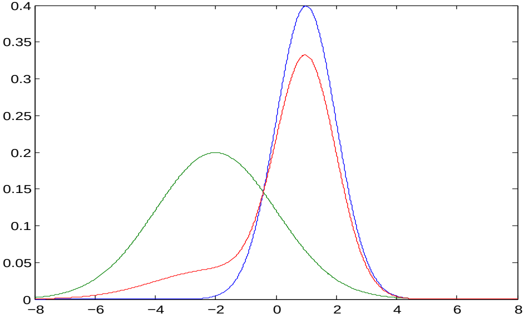
- Regime switches in financial markets:
  - ✓ financial markets tend to change behavior in an abrupt way;
  - ✓ the new behavior may be temporary but often it lasts for several periods after such a change.
- Regime-switching models are well suited to capture this behavior.
- They are statistical models that allow the characteristics of time series to vary through time with regimes of more or less persistence.
- Although regimes are identified through a statistical filtering procedure, they can often be interpreted in terms of economic cycles, changes in monetary policy, regulatory changes or other important economic events such as financial crises.

Empirical findings on financial asset returns

- Risk premia, variances, correlations of financial asset return are time-varying.

  - ✓ Equity premia: risk or price of risk, or both, vary.
  - ✓ Conditional heteroscedasticity: alternation of periods of high volatility followed by periods of low volatility.
  - ✓ Conditional correlations across assets are time-varying: stocks and bonds (positive and negative); international stock markets (much higher correlation in bear markets).
  - ✓ Therefore, unconditional distributions of asset returns are often skewed and fat-tailed.

Mixture of Normal Distributions



Asymmetric (1.70) and fat-tailed (4.6) distribution.

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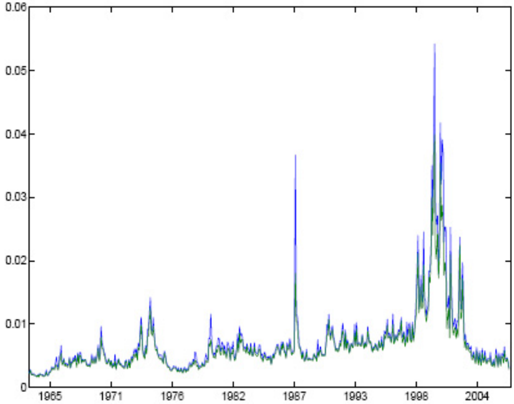
Volatility and Regime Changes

- We will introduce Markov-switching models with a simple two-regime volatility model.
- We will measure volatility by the cross-sectional variance of stock returns. This measure is strongly correlated with an option-implied measure such as the VIX.
- For our purpose it is a simple and model-free measure, applicable at all frequencies.
- This measure is time-varying, with abrupt changes at times.

Cross-sectional variance of stock returns (US)

Source: Mantilla, Garcia and Martellini, 2010



Volatility across time

- The level of volatility is time-varying.
- The volatility of volatility also changes with time.
- Variance risk premium is linked to this volatility of volatility.

Regime-switching models

- Suppose we assume the existence of two regimes in the level and volatility of volatility.
- Regime 1: high level and high volatility, Regime 2: low level and low volatility.
- A statistical model provides for each point in the sample the probability of belonging to regime 1 or regime 2.
- This probability depends on the observed volatility and on the transition process from one state to the other.
- At each date, we suppose that there is a constant probability to stay in the current regime, different for each regime.

The model and the parameters to estimate

The model

$$x_t - \mu_i = \phi(x_t - \mu_j) + \sigma_i e_t,\ i,j \in \{1,2\}$$

The parameters

- The level μ_1 and volatility σ_1 of volatility in regime 1.
- The level μ_2 and volatility σ_2 of volatility in regime 2.
- The autorrelation parameter ϕ.
- The probability p of staying in regime 1 and the probability q of staying in regime 2.

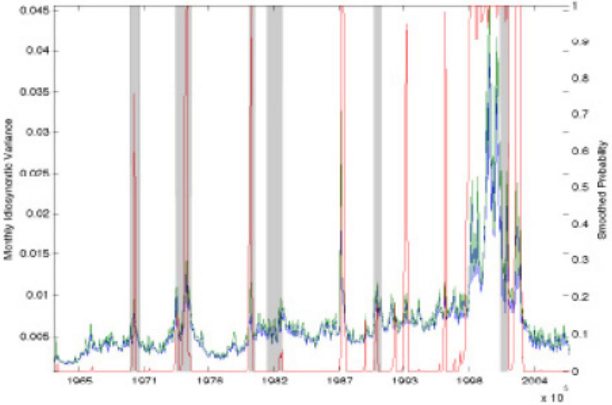
Estimated parameters

Monthly series	CSV^{EW}	CSV^{CW}
μ_1	0.401	0.107
μ_2	0.299	0.065
σ_1	0.067	0.029
σ_2	0.010	0.004
ϕ	0.980	0.839
p	0.839	0.857
q	0.963	0.980

Other useful information from the estimated model

- How long you stay in a regime after you enter it (CSVCW): $\frac{1}{1-p}$
  - Regime 1: 7 months.
  - Regime 2: 50 months.
- Overall percentage of sample in each regime (CSVCW): $\pi_2 = \frac{1-p}{2-p-q}; \pi_1 = 1 - \pi_2$
  - Regime 1: 12.3%.
  - Regime 2: 87.7%.
- Filtered or smoothed probabilities indicate to which regime the sample points belong.

Regime probabilities and recessions



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International Asset Allocation in presence of regime switches

- In bear markets, correlations between international equity returns are higher than during bull markets. Return volatilities are also higher.
- Will investors display home-biased asset choices since diversification benefits are not forthcoming when needed?
- Introducing regimes to capture asymmetric correlations may lead investors to change radically their portfolios across regimes. Is it the case?
- With regime-switching models, you can forecast the probabilities of future regimes and therefore build hedging demands especially for longer horizons. Are these intertemporal hedging demands important?

A US investor looking for international diversification

- The investor maximizes the expected utility of his terminal wealth and has a constant risk aversion.
- He has access to a safe asset with a constant interest rate.
- The problem is solved numerically by dynamic programming.

A US-UK two-regime model in volatilities and correlations

		Estimates	Standard Deviation
	P	0.8546	0.0698
	Q	0.9818	0.0100
US	μ_1	1.1613	0.2198
	μ_2	$= \mu_1$	
	σ_1	7.5064	0.9515
	σ_2	3.7917	0.1654
UK	μ_1	1.2488	0.3090
	μ_2	$= \mu_1$	
	σ_1	14.0748	1.8432
	σ_2	5.2470	0.2409
	ρ_1	0.6181	0.1032
	ρ_2	0.4480	0.0491

Source: Ang and Bekaert (2002), Table 1.

Equity Weights with a Constant Risk-Free Rate Asset

	Regime 1		Regime 2	
	US	UK	US	UK
1	0.2785	0.1039	0.8621	0.4175
12	0.2766	0.1038	0.8613	0.4173
36	0.2762	0.1037	0.8612	0.4173
60	0.2762	0.1037	0.8612	0.4173

IID weights	US	UK
	0.5267	0.2067

Source: Ang and Bekaert (2002), Table 7.

The economic costs of following non-optimal strategies

- This is the compensation (cents per dollar) required for an investor to hold non-optimal portfolios.
  - Costs of non international diversification

T	Regime 1	Regime 2
1 year	0.64	0.91
5 years	4.14	4.46

  - Costs of ignoring regime switching

T	Regime 1	Regime 2
1 year	1.71	1.04
5 years	6.48	5.64

  - Cost of using myopic strategies: negligible.
- The cost of non-international diversification increases when adding other international indices.
- The cost of ignoring regime switching nearly triples when interest rate predicts returns, switches between regimes and affects transition probabilities ($p_{ii,t} = \frac{exp(a_i+b_i r_t)}{1+exp(a_i+b_i r_t)}$).

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Classical asset allocation between stocks, bonds and T-bills

- We consider an investor that allocates his wealth between stocks (large and small capitalization), bonds and Treasury bills.
- He maximizes the utility of his terminal wealth.
- We model the return distribution with a single regime and a four-regime switching model.
- We solve the problem analytically and find good approximate solutions with mean-variance terms and intertemporal hedging demands.

Estimates for the single-regime model, monthly, 1962-2011

Panel A: Single State Model	Large Caps	Small Caps	LT Bonds
Mean Returns	0.89%	1.14%	0.53%
Volatilities/Correlations - Large Caps	0.0420		
Small Caps	0.7431	0.0605	
LT Bonds	0.1136	-0.0030	0.0214

- The optimal portfolio for an investor with a risk aversion of 5 is given by:

Large stocks	18%
Small stocks	29.5 %
LT Bonds	41%
T-bills	11.5%

A four-regime model for stocks, bonds and T-bills, monthly, 1962-2011

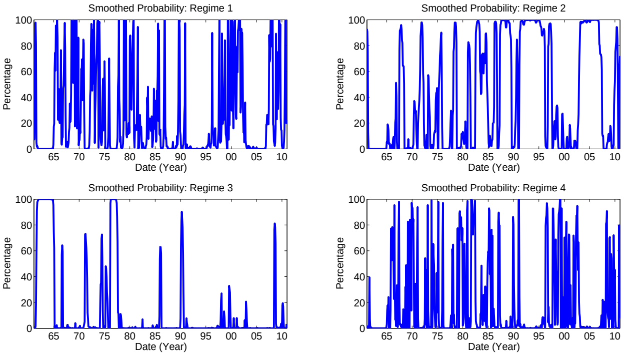
- We consider a four-regime model that characterizes well the return distribution of stocks (large and small capitalization), long-term bonds and Treasury bills.
  - Regime 1: bear
  - Regime 2: slow growth
  - Regime 3: bull market
  - Regime 4: recovery
- In these regimes volatilities and correlations vary considerably in magnitude. Correlations also change signs.

Parameter estimates - Four-regime model, monthly, 1962-2011

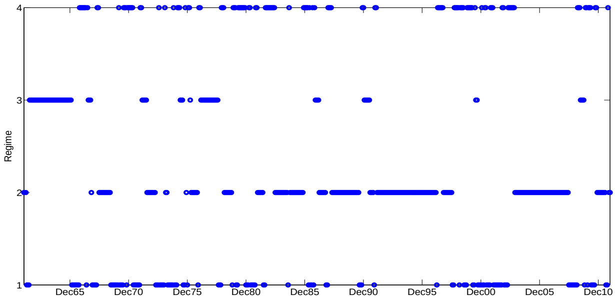
Panel B: Four State Model	Large Caps	Small Caps	LT Bonds
Mean Returns - Regime 1 (Bear)	-2.38%	-3.25%	0.59%
Regime 2 (Slow Growth)	1.14%	0.86%	0.38%
Regime 3 (Bull)	1.40%	2.49%	0.25%
Regime 4 (Recovery)	3.91%	6.42%	1.09%
Volatilities/Correlations - Regime 1 (Bear)			
Large Caps	0.0516		
Small Caps	0.8302	0.0801	
LT Bonds	-0.0267	0.0061	0.0255
Volatilities/Correlations - Regime 2 (Slow Growth)			
Large Caps	0.0295		
Small Caps	0.7146	0.0366	
LT Bonds	0.0813	-0.0447	0.0174
Volatilities/Correlations - Regime 3 (Bull)			
Large Caps	0.0320		
Small Caps	0.7370	0.0424	
LT Bonds	0.3559	0.0875	0.0052
Volatilities/Correlations - Regime 4 (Recovery)			
Large Caps	0.0373		
Small Caps	0.1192	0.0516	
LT Bonds	0.3303	-0.1274	0.0305

Transition Probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1 (Bear)	66.11%	6.15%	4.74%	22.06%
Regime 2 (Slow Growth)	0.001%	93.85%	6.26%	14.19%
Regime 3 (Bull)	0.71%	0.00%	88.78%	7.13%
Regime 4 (Recovery)	33.17%	0.00%	0.22%	56.61%

Smoothed Probabilities of Being in each Regime



Average persistence and duration of regimes



- Average persistence : $1/(1-p_{ii})$
- Regime 1: 3 months; Regime 2: 16 months; Regime 3 : 9 months; Regime 4 : 2.5 months.
- Overall time spent in each regime in the sample: 1. 21.5%, 2. 50%, 3. 12%, 4. 16.5%

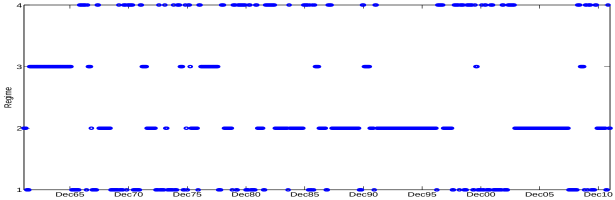
Intuitions on regimes and asset allocation

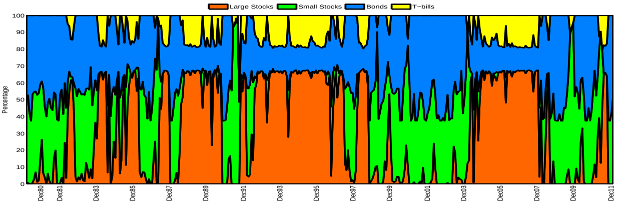
- The relation between the investor horizon of a buy-and-hold strategy and the optimal portfolio varies considerably from one regime to the other.
- For example, in a bear regime, stocks are less favored and short-term investors allocate a smaller part of their portfolio to stocks.
- On the contrary, in the longer run, there is a high probability to switch to a better regime and long-term investors dedicate a larger part of their portfolio to stocks.
- In a bear regime the share allocated to stocks increases with the investor's horizon.

Intuitions on regimes and asset allocation

- However, in more persistent slow-growth or bull regimes, short-term investors allocate larger shares of their portfolio to stocks.
- At longer horizons, investment opportunities will most probably deteriorate, so investors hold less stocks.
- In these regimes, the stock holdings decrease as the horizon lengthens.

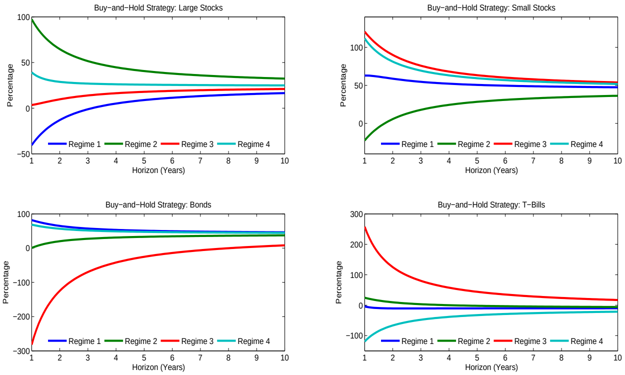
Shares of the various asset classes in the short-term





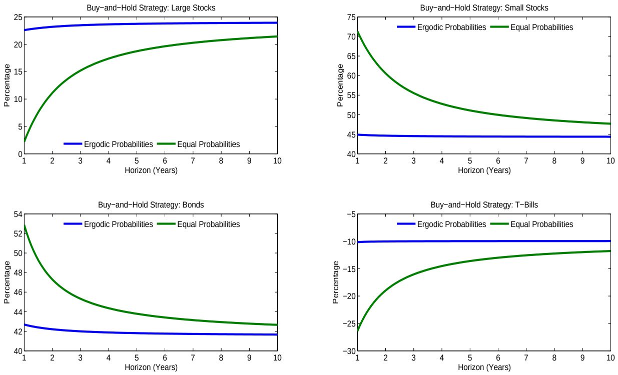
Short selling is **NOT** allowed

Shares in the various asset classes as a function of the horizon



Short selling is allowed and the initial state is known.

Shares in the various asset classes as a function of the horizon



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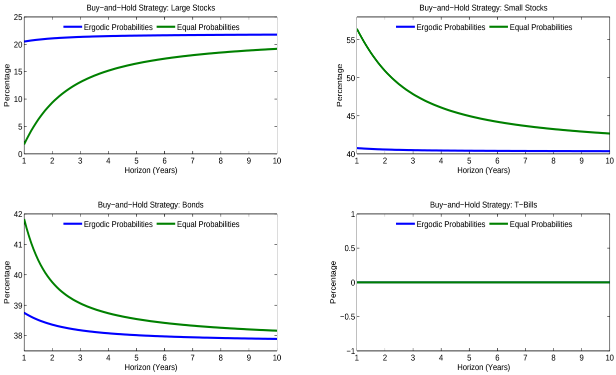

Short selling is **NOT** allowed and the initial state is unknown.

Allocation with regimes and other predictors

- Several studies have uncovered predictors for stocks and bond returns.
- What happens to the optimal portfolio if say the dividend yield is added to a regime-switching model?
- What differences regimes make with respect to a linear model with the dividend yield as predictor?

A model with regime-switching and predictability by the dividend yield

$$\begin{pmatrix} r_t \\ dy_t \end{pmatrix} = \begin{pmatrix} \mu_{r,s_t} \\ \mu_{dy,s_t} \end{pmatrix} + \Phi_{s_t} \begin{pmatrix} r_{t-1} \\ dy_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{r,t} \\ \varepsilon_{dy,t} \end{pmatrix}$$

with:$(\varepsilon_{r,t}, \varepsilon_{dy,t}) \sim \mathbb{N}(0, \Omega_{s_t})$

- The relationship between asset returns and the dividend yield is linear within each regime.
- However, the model is able to capture a nonlinear relationship since the coefficients of the dividend yield change with the regime.

Fundamental intuitions about predictability by the dividend yield and regimes

- Predictability by the dividend yield in an autoregressive linear model reduces risk at longer horizons and induces an increased demand for stocks, the longer the investment horizon.
- This is due to a mean-reverting effect in returns: a negative return shock implies a higher dividend yield and therefore higher future expected returns.
- With regimes, a negative shock is associated with future probable bad states and so the risky asset demand will decrease with the investment horizon.
- When regime switching is combined with predictability, the effect of regimes tends to dominate at short horizons while the mean reverting component in returns tracked by the dividend yield dominates at longer horizons and leads to an increased demand for stocks.

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Summary

- Regime-switching models are able to characterize well the joint distribution of financial asset returns.
- The presence of regimes to capture the distribution of asset returns has important consequences for asset allocation.
- Effects vary according to the regime and the investment horizon.
- The share invested in stocks may decrease with the horizon contrary to a linear model with predictability and mean reversion.
- Predictability changes asset allocation even with regimes, especially at long horizons.