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Foreword

The present publication is part of the BNP Paribas Investment Partners research chair at EDHEC-Risk Institute on "ALM and Institutional Investment Management". This chair, under the supervision of Professor Lionel Martellini, examines the properties of dynamic asset allocation strategies in asset-liability management. Over the past few years, this chair has already given rise to two important publications. Among many other useful insights, this research has emphasised that capital structure decisions made by the sponsor and allocation decisions made by the pension fund can and should be jointly analysed within the context of an integrated ALM model. Moreover, it has shown that dynamic risk-controlled investment strategies are an effective way to align the incentives of shareholders and pensioners without any complex or costly adjustment to the pension plan structure.

The purpose of the present research publication is to go beyond simple forms of dynamic strategies, and to show that more sophisticated dynamic allocation strategies could usefully be implemented by pension funds. For instance, the paper shows that imposing a cap on the funding ratio, in addition to a floor, has a positive impact on both pensioners and bondholders, while only having a minor negative effect on equity value. The paper also introduces novel forms of dynamic strategies that recognise that pension risk is not only driven by the funding ratio of the pension fund, but also by the financial strength or weakness of the sponsor company. These strategies aim to control sponsor risk by avoiding states of the world where the pension fund is underfunded and the sponsor is unable to make up for the gap.

I would like to thank the co-authors, Lionel Martellini, Vincent Milhau and Andrea Tarelli, for the quality of their research. I would also like to thank our partners at BNP Paribas Investment Partners for their support and for their continuous commitment to this research chair.

We wish you a pleasant and informative read.

Noël Amenc
Professor of Finance
Director of EDHEC-Risk Institute
About the Authors

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Summary of Results
## Summary of Results

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1. Introduction
1. Introduction

Previous research has emphasised the benefits of dynamic risk-controlled allocation strategies in asset-liability management (ALM). Detemple and Rindisbacher (2008) consider a pension fund which can be either defined-benefit (DB) or defined-contribution (DC), and they derive optimal portfolio and cash extraction policies. When preferences are expressed over the terminal “partial surplus” (see Sharpe and Tint 1990) and the pension fund has constant relative risk aversion (CRRA) utility, the optimal strategies extend some commonly-used forms of dynamic strategies used in an asset-only context, such as CPPI strategies (see Black and Jones 1987; and Black and Perold 1992), to the ALM context. In a subsequent paper, Martellini and Milhau (2010b) analyse the optimal allocation policy for a pension fund facing regulatory constraints on the funding ratio in a continuous-time model with uncertain interest and inflation rates. As a function of the outstanding risk budget, the solution involves a dynamic allocation to two building blocks – a “risky” and a “safe” building block. The risky block coincides with the strategy that would be optimal in the absence of funding constraint (which is very different from a traditional CPPI strategy, where the risky block is typically a stock index); meanwhile the safe block is the asset portfolio that has the highest correlation with liabilities (this portfolio is known as liability-hedging portfolio, or LHP in short). Insurance against downside risk has a cost, which can be measured by the loss of access to the upside performance of the unconstrained strategy: the higher the minimum funding level protected, the lower the access to upside, because a large fraction of wealth is used to “purchase” insurance against downside risk (see MM2010b for more details).

Other forms of dynamic portfolio strategies can be relevant for pension funds. For DC pension funds, the wide literature on life-cycle investing provides examples of optimal strategies. For example, Teplá (2001) derives an optimal strategy that is similar to the strategies analysed in MM2010b, while Basak et al. (2006) relax the constraint that the portfolio must outperform the benchmark with probability one: instead of imposing a constraint of the “almost sure” type, as in Teplá (2001) and MM2010b, they impose a constraint “in probability”, which states that the probability of underperforming the benchmark must not exceed a fixed (small) level. Other examples can be found outside the paradigm of expected utility. An example is given by Föllmer and Leukert (1999), who compute strategies that minimise the shortfall probability or the expected shortfall for an investor who wishes to attain a target wealth level, but starts with an initial capital that is less than the present value of the target. By absence of arbitrage opportunities, it is impossible to ensure that the target will be attained in all states of the world, but the agent can still attempt to maximise the probability of reaching the target, or to minimise the expected size of the shortfall.

In all these papers, however, the asset allocation problem for the pension fund is analysed in isolation from the sponsor company. Martellini and Milhau (2010a) (henceforth MM2010a) introduce a more integrated approach to asset-liability management, with a focus on optimal allocation decisions at the pension fund
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level, as well as optimal contribution policies at the sponsor level. Within the context of a formal capital structure model, they point out the benefits related to the use of relatively basic forms of dynamic allocation strategies – in fact, simple CPPI strategies. This paper is an attempt to provide a thorough analysis of a wide range of dynamic risk-controlled strategies within an integrated ALM framework. Among other things, we consider (i) strategies involving a floor given as a function of the regulatory and/or liability portfolio value; (ii) strategies involving a performance cap in addition to floors so as to allow for a decrease in the cost of downside risk protection; (iii) strategies involving corporate bonds, as opposed to Treasury-bonds, in the liability hedging portfolio; and (iv) strategies based on risk controls that encompass state variables related to the sponsor company, so as to better focus on hedging away those states of the world characterised by a joint occurrence of poorly performing pension assets and weak financial health on the sponsor company side.

The results we obtain suggest that enlarging the set of admissible investment strategies so as to include dynamic risk-controlled strategies proves to be an effective way to align the incentives of shareholders, who naturally benefit from risk taking, and pensioners, who typically do not have access to surpluses and therefore have little interest, if any, in risk/return-enhancing strategies. More specifically, our findings can be summarised as follows. First, we find that implementing even relatively basic forms of risk-controlled strategies aiming at insuring a minimum funding ratio level above a minimum value allows shareholders to get some (limited) access to the upside performance of risky assets, while ensuring that pensioners will not be overly hurt by the induced increase in risk. We also find that imposing a cap on the terminal funding ratio, as was done in MM2010b, allows for higher access to the upside performance of risky assets, when such assets deliver low to medium performance. As a result, this strategy has a positive impact on pensioners and bondholders, who have either no access or limited access to pension fund surpluses. This comes at the cost of a decrease in equity value, because the strategy precludes the possibility of large surpluses, but the upside potential remains substantial from the perspective of equity holders. Finally, we test strategies that aim at controlling sponsor risk by providing insurance against states of the world characterised by the joint occurrence of an underfunded pension plan and a weak sponsor company. Since by definition, such states of the world are fewer than those characterised solely by an underfunded pension plan, we find that the cost of downside risk insurance has decreased compared to what is achieved with more basic dynamic strategies. Such strategies are particularly relevant for initially underfunded pension plans, as they avoid those states of the world where the pension fund ends up underfunded and the sponsor firm does not have enough available assets to make up for the deficit. In fact, under some circumstances, these strategies are found to increase welfare for all stakeholders, and we discuss some practical challenges related to real-world implementation of these otherwise attractive strategies. Overall, our findings suggest that dynamic portfolio strategies can prove to be a very effective answer to some key challenges currently faced by corporate pension plans.
1. Introduction

Following related literature, our analysis of the allocation problem for pension funds is cast in a simplified setting where default and additional contributions can happen only at terminal date. In practice, of course, pension plan sponsors are required by the regulator in most countries to make additional contributions at intermediate dates, unless funding ratio levels are sufficiently high, in which case they benefit from “contribution holidays”. While an extension of our results to account for the presence of intermediate contributions from the sponsor would be worthwhile, we believe that the main qualitative features of the problems are captured in the static setting. If anything, we expect the benefits of dynamic risk-controlled strategies to be further enhanced in a setting with required intermediate contributions, since such strategies would allow the shareholders of pension plan sponsors to dynamically manage the risk of underfunding, and as such to benefit from contribution holidays if minimum funding ratios are set at corresponding levels.

The rest of the paper is organised as follows. In section 2, we describe the financial market and we introduce the notations, and section 3 presents the integrated ALM model. Section 4 analyses the benefits of CPPI strategies in an integrated ALM context. Section 5 presents several extensions of these strategies. In section 6, we study strategies that aim at avoiding states of the world where the pension fund ends up underfunded and the sponsor is unable to make up for the deficit. Section 7 concludes.
2. The Economy
2. The Economy

In this section, we introduce the formal model for the financial market. Uncertainty in the economy is represented by a standard probability space \((\Omega, \mathcal{F}, \mathbb{P})\), where \(\mathbb{P}\) is a probability measure that represents the assessment by the pension fund of the likelihood of future events, and \(\mathcal{F}\) is a sigma-algebra that consists of all measurable events. The finite time span is denoted with \([0, T]\), where \(T\) is the horizon of the pension fund.

2.1 Financial Variables and Traded Assets

The framework is that of MM2010b. The nominal short-term interest rate, \(r\), follows a mean-reverting process (Vasicek 1977):

\[\text{d}r_t = a(b - r_t)\text{d}t + \sigma^r \text{d}z^r_t,
\]

where \(z^r\) is a Brownian motion defined on the probability space \((\Omega, \mathcal{F}, \mathbb{P})\).

The price index, \(\Phi\), follows a Geometric Brownian motion:

\[\frac{\text{d}\Phi_t}{\Phi_t} = \pi \text{d}t + \sigma^\Phi \text{d}z^\Phi_t,
\]

where \(z^\Phi\) is also a Brownian motion, that has a correlation coefficient \(\rho^{\Phi r}\) with \(z^r\).

The pension fund can invest in a cash account, whose value at date \(t\) is the continuously compounded interest rate:

\[S^0_t = e^{\int_0^t r_s \text{d}s}.
\]

This asset has zero duration, which means that the change in its value over a small interval \([t, t + \text{d}t]\) is known as of date \(t\) and has zero covariance with changes in the interest rate. This can be mathematically seen by writing that

\[\text{d}S^0_t = S^0_t r_t \text{d}t.
\]

In order to hedge interest rate risk, the pension fund needs to have access to a security with positive duration. An example of such an instrument is a zero-coupon bond paying $1 at date \(T\), the price of which at date \(t\) is given by the standard formula:

\[B(t, T) = e^{-D(T-t)r_t + E_n(T-t)},
\]

where \(D(T-t)\) is the duration of the bond (i.e. its sensitivity with respect to the short-term rate) and \(E_n\) is another function of the time-to-maturity. \(D\) and \(E_n\) are given by:

\[D(T-t) = \frac{1 - e^{-\alpha(T-t)}}{\alpha},
\]

\[E_n(T-t) = \left(b - \frac{\sigma^r \lambda^r}{\alpha}\right)[D(T-t) - (T-t)] + \frac{(\sigma^r)^2}{2\alpha^2} \left[T-t - 2D(T-t) + \frac{1 - e^{-2\alpha(T-t)}}{2\alpha}\right].
\]

The parameter \(\lambda^r\) is the market price of interest rate risk. The dynamics of the bond price follows from Ito’s lemma:

\[\frac{\text{d}B(t, T)}{B(t, T)} = [r_t - D(T-t)\sigma^r \lambda^r] \text{d}t - D(T-t)\sigma^r \text{d}z^r_t. \tag{2.1}
\]

Since the model has one factor only, this zero-coupon bond as well as any other one with a different maturity date can be replicated by trading dynamically in cash and in a bond with constant maturity \(\tau\), the price of which evolves as:

\[\frac{\text{d}B_t}{B_t} = [r_t - D(\tau)\sigma^\tau \lambda^\tau] \text{d}t - D(\tau)\sigma^\tau \text{d}z^\tau_t.
\]

In what follows we will assume that the pension fund has access to a nominal zero-coupon maturing at the horizon \(T\), which can be replicated using a constant-maturity bond if not readily available on the market. We shall refer to this zero-coupon simply as the nominal bond.
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In addition to nominal bonds, the pension fund can also trade in indexed bonds. An indexed zero-coupon bond of maturity \( T \) pays \( \Phi_T \) at date \( T \). As shown by MM2010b, if the price of inflation risk, \( \lambda \Phi \), is constant, then the price of this bond at a date \( t \) is given by:

\[
I(t, T) = \Phi_t e^{-r(T-t)} + E_t(T-t),
\]

where:

\[
E_t(T-t) = (\pi - \sigma^* \lambda^*) (T-t)
\]

\[
+ \left( b - \frac{\sigma^* (\lambda^* - \sigma^* \rho^*)}{\alpha} \right) D(T-t) - (T-t) \]

\[
+ \frac{(\sigma^*)^2}{2\alpha^2} \left[ T - t - 2D(T-t) + \frac{1 - e^{-2\alpha(T-t)}}{2\alpha} \right].
\]

Applying Itô’s lemma to the process \( I(\cdot, T) \), we obtain the dynamics of the indexed bond price:

\[
\frac{dI(t, T)}{I(t, T)} = \left[ r_t - D(T-t)\sigma^* \lambda^* + \sigma^* \Phi_t^* \right] dt
\]

\[
- D(T-t)\sigma^* \lambda^* dz_t^* + \sigma^* dz_t^*.
\]

As for the nominal bond, the dynamics of a constant-maturity indexed bond reads:

\[
\frac{dI_t}{I_t} = \left[ r_t - D(\tau)\sigma^* \lambda^* + \sigma^* \Phi_t^* \right] dt
\]

\[
- D(\tau)\sigma^* \lambda^* dz_t^* + \sigma^* dz_t^*.
\] (2.3)

The indexed zero-coupon with fixed maturity date \( T \) can be replicated by dynamically trading in the constant-maturity indexed bond, the constant-maturity nominal bond and the cash account. It should be noted that trading only in the indexed bond and cash does not allow for perfect replication of the vanishing-maturity indexed bond. Indeed, as is clear from the dynamics (2.3), all indexed bonds have the same exposure to inflation risk, but they differ through their exposures to interest rate risk. As a consequence, one must be able to separately control the exposures to both risks in order to replicate a vanishing-maturity indexed bond. This cannot be achieved by trading in the constant-maturity indexed zero-coupon only, but can be done by trading in both constant-maturity bonds.

Finally, the pension fund can also trade in a stock index \( S \), the price of which evolves as:

\[
\frac{dS_t}{S_t} = \left[ r_t + \sigma S^* \lambda S^* \right] dt + dz_t^S.
\]

The Brownian motion \( z^S \) has correlation \( \rho^S \) with \( z^r \) and \( \rho^S \Phi \) with \( z^\Phi \).

The last stochastic process in our model is the unlevered asset value of the sponsor company, for which we assume the following dynamic evolution:

\[
\frac{dV_t}{V_t} = \left[ r_t + \sigma^V \lambda^V \right] dt + \sigma^V dz_t^V, \tag{2.4}
\]

where \( \lambda^V \) is a constant expected excess return, the volatility \( \sigma^V \) is constant, and \( z^V \) is a fourth Brownian motion correlated with the other Brownian motions of the model. A possible interpretation of \( V \) is the present value of future earnings of the company (see Goldstein et al. (2001)). In practice, the source of risk \( z^V \) could be hedged by dynamically trading in the equities of the sponsor plan, as well as in bonds and in the stock index. Indeed, as will be argued below (see subsection 3.3), the value of equities is a function of time and of the current value of the sponsor’s assets, interest rate, price index and price of stock index. Because they are only imperfectly correlated with \( V \), equities cannot by themselves hedge away the source of risk \( z^V \), and they have to be incorporated in a more complex dynamic trading strategy involving the other risky assets. Unless otherwise explicitly stated, we
will assume in this paper that the pension fund cannot trade in the equities of its own sponsor plan, and therefore has no access to a security perfectly correlated with $V$. As a consequence, the market is incomplete from the pension fund’s perspective. Only in section 6 below will we assume that the pension fund can either perfectly hedge away $zV$ by trading in the sponsor’s equities, or can at least partially hedge away that risk by trading in a proxy that is highly correlated with $V$.

### 2.2 Introducing A Defaultable Corporate Bond

In some cases (e.g. subsection 5.3 below, on the control of regulatory funding ratio), one may want to assume that the pension fund can also invest in a corporate bond subject to default risk. Of course, in our model, the bonds issued by the sponsor are defaultable (see subsection 3.3 below), but their valuation is endogenous, and their price depends on all the parameters of our model, including the allocation strategy taken by the pension fund. Bonds of the sponsor company are therefore difficult to handle as another asset class.

A different approach is to introduce a defaultable bond issued by another company. For tractability purposes, we assume that the default of this bond occurs at a random date $\Theta$ that is exponentially distributed with parameter $\nu^\mathbb{Q}$ under $\mathbb{Q}$, and exponentially distributed with parameter $\nu^\mathbb{P}$ under $\mathbb{P}$. The $\mathbb{P}$-intensity has to be larger than the $\mathbb{Q}$-intensity, for the model to be able to reproduce the following well-documented stylised fact about historical probabilities of default: they are smaller than the probabilities implied by observed credit spreads, which can be reproduced by the model if one assumes $\nu^\mathbb{P} > \nu^\mathbb{Q}$. In our numerical illustration with a corporate bond in subsection 5.3, we will take $\nu^\mathbb{P} = \frac{\nu^\mathbb{Q}}{2}$.

We will assume for simplicity that the bond has zero recovery, and that the default date is independent from the processes $r$, $\Phi$, $V$ and $S$. The price of a corporate indexed bond that promises the payoff $\Phi_T$ at date $T$ is then given by (see Lando 2004):

$$X_t = 1_{\{\tau > \Theta\}} e^{-\nu^\mathbb{Q}(T-t)} I(t, T).$$

This equation means that the price of the bond equals the expectation of the face value discounted at a rate higher than the risk-free rate, with the spread being equal to the default intensity under $\mathbb{Q}$.

To summarise, the pension fund can trade in cash, in a nominal zero-coupon maturing at date $T$, an indexed bond maturing with the same maturity date, the stock, and, if explicitly indicated, a corporate bond or a security that is highly or even perfectly correlated with $V$.

### 2.3 Pricing Kernel

For notational simplicity, it is useful to rewrite the model in vector form. We let $z$ be a 4-dimensional Brownian motion, and $\sigma^r$, $\sigma^\Phi$, $\sigma^S$ and $\sigma^V$ be the corresponding volatility vectors of $r$, $\Phi$, $S$ and $V$, that satisfy:

$$\sigma^r dz^r_t = (\sigma^r)' dt, \quad \sigma^\Phi dz^\Phi_t = (\sigma^\Phi)' dt, \quad \sigma^S dz^S_t = (\sigma^S)' dt, \quad \sigma^V dz^V_t = (\sigma^V)' dt.$$
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The volatility vectors of the vanishing-maturity bonds are thus:

\[ \sigma^B(T-t) = -D(T-t)\sigma^r, \]

\[ \sigma^I(T-t) = -D(T-t)\sigma^r + \sigma^\Phi, \]

The volatility matrix of traded assets is the matrix whose columns are the volatility vectors of the traded assets. It is time-dependent because of the vanishing maturities of the bonds:

\[ \sigma_t = \begin{pmatrix} \sigma^S & \sigma^B(T-t) & \sigma^I(T-t) \end{pmatrix}. \]

This matrix can be also expressed as the product of a "unit" volatility matrix \( \rho \) and the matrix of loadings of the assets on the traded risks:

\[ \sigma_t = \rho Q_t, \]

where:

\[ \rho = \begin{pmatrix} \sigma^r & \sigma^b & \sigma^\Phi \\ \sigma^S & -D(T-t)\sigma^r & -D(T-t)\sigma^r \\ 0 & 0 & \sigma^\Phi \end{pmatrix}, \]

\[ Q_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -D(T-t)\sigma^r & -D(T-t)\sigma^r \\ 0 & 0 & \sigma^\Phi \end{pmatrix}. \]

The associated market price of risk vector is:

\[ \lambda = \rho \rho^{-1} \begin{pmatrix} \lambda^V \\ \lambda^S \\ \lambda^\Phi \end{pmatrix}. \]

It is to be distinguished from the price of risk vector implied by the market, \( \Lambda \), which is defined as:

\[ \Lambda = R (R' R)^{-1} \begin{pmatrix} \lambda^V \\ \lambda^S \\ \lambda^\Phi \end{pmatrix}, \]

where \( R \) is the matrix that collects the normalized volatility vectors:

\[ R = \begin{pmatrix} \sigma^r & \sigma^\Phi & \sigma^S & \sigma^V \\ \sigma^r & \sigma^\Phi & \sigma^S & \sigma^V \end{pmatrix}. \]

With the previous notations, the unique pricing kernel of the economy is defined as:

\[ M_t = \exp \left[ -\int_0^t r_s \, ds - \frac{1}{2} \int_0^t \| \Lambda \|^2 \, ds - \int_0^t \Lambda' dz_s \right], \quad t \leq T. \]

The associated equivalent martingale measure (EMM) is defined by:

\[ \frac{dQ}{dP} = \exp \left[ -\int_0^T \| \Lambda \|^2 \, ds - \int_0^T \Lambda' dz_s \right]. \]

Girsanov's theorem then shows that the process \( z_t^Q \) defined by

\[ z_t^Q = z_t + \Lambda t \]

is a \( Q \)-Brownian motion.

The martingale measure \( Q \) is the one that is given by the market, so the prices of the claims written on the assets of the sponsor and the pension fund are computed as expected discounted values under this measure. It should be noted that the pension fund faces an incomplete market situation, because it does not have access to a security that fully hedges away the source of risk \( z^V \). Hence it may choose a pricing measure different from \( Q \) when performing a portfolio optimisation exercise (see appendix A.2).
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3. An Integrated ALM Model
3. An Integrated ALM Model

In this section, we describe the budget constraint under which the pension fund operates, the promised pension liabilities, and the actual payoffs to the various present stakeholders, who are pensioners, equity holders and corporate debtholders.

3.1 Pension Fund Budget Constraint

As explained above, the pension fund can trade in cash and in three locally risky assets, namely the stock and two zero-coupon bonds, possibly one corporate bond and one security that hedges sponsor risk. In fact, for most of this paper, we will assume that the traded assets include only cash, the stock and the two zero-coupon bonds. The only exceptions are subsection 5.3 and section 6. Thus, to alleviate the notations, we define \( w_t \) to be the vector of weights allocated to the three locally risky assets at date \( t \). In case another asset is traded, \( w_t \) will be extended to account for its presence in the investment universe. By convention, the elements of \( w_t \) are ordered as follows:

\[
w_t = \begin{pmatrix}
\text{stock index} \\
\text{nominal bond} \\
\text{indexed bond}
\end{pmatrix}.
\]

Over the period \([0, T]\), the pension fund manages a self-financing portfolio, which means that it does not receive any contribution from the sponsor after date 0, and pays nothing to pensioners until date \( T \). Denoting the value of the portfolio with \( A_t \), we can express the budget constraint as: \(^2\)

\[
dA_t = A_t \left[ \left( r_t + w_t' \sigma_t \Lambda \right) dt + w_t' \sigma_t dz_t \right].
\]

The initial wealth \( A_0 \) is the endowment provided by the sponsor company to its pension plan. It is taken out from an initial amount of capital \( x \) that is initially available to the company, and that is split between the pension plan and the operating projects. The amount allocated to these projects is \( V_0 \). Because the contribution to the pension plan is tax-deductible, the initial budget constraint can be written as:

\[
(1 - \theta)A_0 + V_0 = x,
\]

where \( \theta \) is the tax rate. This relationship shows that if the sponsor allocates \( A_0 \) to its pension fund, it can invest more than \( x - A_0 \) in the industrial projects, because the after-tax cost of the contribution is only \((1 - \theta)A_0\).

3.2 Pension Liabilities

Pension liabilities are summarised by a single payment \( L_T \) at date \( T \). In this paper we will consider liabilities that are either unconditionally or conditionally indexed on inflation. First, indexation can be unconditional, in which case the promised payment to pensioners is fixed in real terms, hence \( L_T \) is of the form \( L \Phi_T \) for some fixed \( L \). This will be our base case. The second form of indexation is conditional: the promised payment to pensioners is a fixed nominal amount \( L \), but pensioners actually receive \( L \Phi_T \) if the final funding ratio is in excess of some fixed threshold \( K \) which we will take as equal to 100\%. Conditional indexation is thus a form of surplus sharing, which gives pensioners a capped access to surpluses of the pension fund. In brief, the promised payment to pensioners can be written as:

\[
L_T = \begin{cases} 
L \Phi_T & \text{with unconditional indexation,} \\
L & \text{with conditional indexation.}
\end{cases}
\]
3. An Integrated ALM Model

We denote with $L_t$ the present value at date $t$ of the promised payment $L_p$. The value $L_t$ must be distinguished from the fair value of the liability payments, which takes into account, through an endogenous, plan-specific, credit spread adjustment, the existence of default risk on these payments in case of the joint occurrence of a weak sponsor company and under-funded pension plan. It should also be distinguished from the regulatory value, which is obtained by discounting the promised payoff at the short-term rate plus an arbitrary credit spread $s_{\text{reg}}$, the same for all pension plans regardless of their funding status and the health of their sponsor company. International accounting standards SFAS 87.44 and IAS19.78 recommend the use of the market yield on AA corporate bonds in order to value pension obligations. For simplicity, we assume this spread to be constant, so we have:

\[
\begin{align*}
L_t &= L(t, T) \\
L_t^{\text{req}} &= L e^{-s_{\text{reg}}(T-t)}(t, T) \\
L_t &= LB(t, T) \\
L e^{-s_{\text{reg}}(T-t)B(t, T)}
\end{align*}
\]

with unconditional indexation.

It should be noted that the regulatory value is smaller than the present value of the promised payment, because the spread is positive. Unless otherwise stated, the funding ratio is the ratio of asset to the present value of promised liabilities $L$:

\[
R_t = \frac{A_t}{L_t}.
\]

while the regulatory funding ratio is the ratio of asset to regulatory value of liabilities:

\[
R_t^{\text{req}} = \frac{A_t}{L_t^{\text{req}}}.
\]

We define a liability-hedging portfolio (LHP) as a portfolio that has perfect instantaneous correlation with the process $(L_t)_t$. If indexation is unconditional, this portfolio is fully invested in an indexed zero-coupon that matures at date $T$, and pays $\Phi_T$. If indexation is conditional, the promised payment is fixed in nominal terms, so the LHP is invested in a nominal zero-coupon that pays 1 at date $T$. With the above notations, we have:

\[
u_t^{\text{LHP}} = \begin{cases} 
    \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, & \text{with unconditional indexation.} \\
    \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, & \text{with conditional indexation.}
\end{cases}
\]

Investing the capital $L_0$ in the LHP leads to the payoff $L_t$ at date $T$: the promised payoff to pensioners is thus replicable. It does not follow that the actual payoff is also replicable. For instance, if the sponsor were never to default, the actual payoff to pensioners with conditional indexation would be equal to $L$ if $A_T/(L \Phi_T)$ is less than the threshold $K$, and $L \Phi_T$ otherwise. Obviously, the nominal zero-coupon does not replicate this payoff. But in fact the sponsor can default, which makes the actual payoff to pensioners ex-ante distinct from the promised one. As will be argued in the next subsection, the actual payoff is a non-linear function of the assets of the firm and pension fund, and is thus not replicable with a simple zero-coupon, either nominal or indexed.

3.3 Payoffs to Stakeholders

We now detail how we explicitly modelled the transfers of money between the sponsor and the pension fund. Formally, we follow MM2010a and we model an externally funded pension plan committed to paying an amount $L_T$ to pensioners at the terminal
date \( T \). The sponsor, on its side, is committed to paying a fixed amount \( D \) to bondholders. The present value of this commitment at date \( t \) is:

\[
D^p_t = DB(t, T).
\]

(3.2)

In case the asset of the pension fund at date \( T \) does not cover the promised payment, a contribution is required from the sponsor. This contribution is equal to \( L_T - A_T \), but its actual amount is capped by the sponsor’s available assets. Since the sponsor company is committed to paying \( D \) to bondholders, the required contribution is paid only if \( A_T + V_T \geq L_T + D \). In that case, corporate debt is also paid back in full. In the opposite case \( (A_T + V_T < L_T + D) \), the sponsor firm is put bankrupt by equity holders, and both pensioners and bondholders receive a recovery payment. If the pension fund is solvent at date \( T \), two situations may occur.

First, if the sponsor can redeem corporate bonds, bondholders are paid the face value and equity holders receive the part of assets that is in excess of pension liabilities plus corporate debt. Second, if the assets of the sponsor are not sufficient to allow for full redemption, pension fund surpluses can be used. MM2010a discuss a variety of surplus sharing rules, that allow equity holders to benefit from the totality or a fraction of these surpluses. In this paper, we focus only on the case where equity holders have full access to surpluses. If the sponsor in insolvent at date \( T \) but we have \( A_T + V_T \geq L_T + D \), then corporate debt can be fully redeemed. If \( A_T + V_T < L_T + D \), the firm is in default and bondholders receive a recovery payment.

\[
P_T = L_T \left[ 1_{\{A_T \geq L_T\}} + 1_{\{A_T < L_T, A_T + V_T \geq L_T + D\}} \right] \\
+ A_T + q(1 - \alpha) V_T 1_{\{A_T < L_T, A_T + V_T < L_T + D\}} \\
+ (L \Phi_T - L_T) 1_{\{A_T \geq L_T\}},
\]

(3.3)

\[
E_T = \left[ A_T + (1 - q)(1 - \alpha) V_T 1_{\{A_T < L_T, A_T + V_T \geq L_T + D\}} \\
+ q(1 - \alpha) V_T 1_{\{A_T < L_T, A_T + V_T < L_T + D\}} \\
+ (1 - q) V_T + A_T - L_T \right] 1_{\{A_T \geq L_T, A_T + V_T < L_T + D\}},
\]

(3.4)

\[
D_T = D \left[ 1_{\{A_T \geq L_T\}} + (1 - q)(1 - \alpha) V_T 1_{\{A_T < L_T, A_T + V_T < L_T + D\}} \\
+ (1 - q) V_T + A_T - L_T \right] 1_{\{A_T \geq L_T, A_T + V_T < L_T + D\}},
\]

(3.5)

(Note that in the case of unconditional indexation, the quantity \( (L \Phi_T - L_T) 1_{\{A_T \geq L_T\}} \) that appears in the payoffs to pensioners and equity holders, is zero in all states of the world).

The fair values of pension claims, equities and corporate bonds are equal to the expected discounted values of these payoffs under the risk-neutral probability measure:

\[
P_0 = \mathbb{E}^Q \left[ e^{-\int_0^T r_s \, ds} P_T \right],
\]

\[
E_0 = \mathbb{E}^Q \left[ e^{-\int_0^T r_s \, ds} E_T \right],
\]

\[
D_0 = \mathbb{E}^Q \left[ e^{-\int_0^T r_s \, ds} D_T \right].
\]
3. An Integrated ALM Model

The sum of these values defines the total value of the firm and pension fund:

\[ v_0 = P_0 + D_0 + E_0. \]

As shown in MM2010a, the total value can be also expressed in terms of the present value of the tax shield and of the bankruptcy costs. Indeed, a direct manipulation of the payoffs (3.3), (3.4) and (3.5) shows that:

\[ P_T + E_T + D_T = A_T + V_T + T S_T - BC_T, \]

(3.6)

where the tax shield and the bankruptcy costs are given by:

\[ T S_T = \theta [D - D_p + (L_T - A_T)^+] 1_{\{A_T + V_T > L_T + D\}}, \]
\[ BC_T = \alpha V_T 1_{\{A_T + V_T < L_T + D\}}. \]

Taking the present values of both sides of (3.6), we obtain an equivalent expression of the total value:

\[ v_0 = A_0 + V_0 + T S_0 - BC_0. \]

As a last notation, we introduce the contribution made by the sponsor to the pension fund at the terminal date. The required contribution is equal to the deficit, \((L_T - A_T)^+\), but it differs from the actual contribution, because the sponsor has limited assets. Let \(C_T\) denote the actual contribution. Examining the previous payoffs, it can be seen that:

\[ C_T = (L_T - A_T)^+ 1_{\{A_T + V_T > L_T + D\}}. \]

We denote with \(C_0\) the fair value of this contribution at date 0, that is, the expected value of the discounted \(C_T\) under \(Q\).
3. An Integrated ALM Model
4. CPPI Strategies in ALM
This section introduces the definition and some important properties of CPPI strategies extended to ALM context.

4.1 Description of Strategies
The objective of these strategies is to guarantee that the final wealth of the pension fund will satisfy:

\[ A_T \geq kL_T \quad \text{almost surely,} \quad (4.1) \]

for some minimum level \( k \). A natural idea is to take \( k \) at least equal to 100\%, since the pension fund is committed to pay \( L_T \) to beneficiaries. But it must be noted that if (4.1) holds, the assumption of no arbitrage opportunities implies:

\[ A_0 \geq kL_0 \quad (4.2) \]

In particular, if \( k \) is greater than 100\%, the pension fund must be more than fully funded at the initial date. If it is not, then it is impossible to ensure that (4.1) will hold almost surely, whatever the investment policy adopted by the pension fund. In an asset-only context, a CPPI strategy aims at ensuring that the terminal wealth \( A_T \) is greater than some minimum level \( k \). It combines two building blocks: a “risky” asset, which is in general a diversified stock index with attractive performance, and a “safe” asset, which is a zero-coupon maturing at date \( T \). CPPI strategies in ALM have a similar form, but the “safe” asset is the asset that is safe from an ALM perspective (i.e. the asset that delivers the payoff that has been promised to pensioners). This asset is a nominal zero-coupon if the promised payment is fixed in nominal terms (as is the case with conditional indexation) or an indexed zero-coupon if the promised payment is fixed in real terms (as with unconditional indexation). Moreover, the “risky” block is not necessarily invested in a stock index only, and it can be any portfolio strategy denoted as \( w^u \), where superscript \( u \) stands for “unconstrained”. For these reasons, such strategies would have to be labelled “extended CPPI strategies”. For simplicity, however, we will refer to them simply as “CPPI strategies” in what follows. In section 6, we further extend the analysis to dynamic ALM strategies that also encompass control variables related to the health of the sponsor company. Such strategies will sometimes be referred to as risk-controlled investing (RCI) strategies, or dynamic integrated LDI strategies, so as to distinguish them from the more basic CPPI ALM strategies that we describe in this section. Even these basic CPPI strategies substantially differ from CPPI strategies that are commonly adopted in practice in that the underlying unconstrained strategy \( w^u \) is not necessarily entirely invested in stocks.

The floor of the CPPI strategies we consider in this section is taken equal to \( kL_0 \), where \( L_t \) is the present value of the promised payment and \( k \) is a minimum terminal funding ratio. In the context of implicit minimum funding ratio constraints, MM2010b show that the risk budget is naturally defined as the distance of current wealth to the floor, \( A_t - kL_t \). The dollar allocation to the unconstrained strategy is equal to a multiplier, \( m \), times the risk budget. As a consequence, the expression for the weight vector is:

\[ w_t = m \left( 1 - \frac{k}{R_t} \right) w^u_t + \frac{k}{R_t} w^{\text{LIP}}_t \quad (4.3) \]
4. CPPI Strategies in ALM

It is shown in appendix A.1 that the wealth generated by this strategy is:

\[
A_t = kL_t + (A_0 - kL_0) \exp \left[ (1 - m) \int_0^t \left( r_s + \frac{m \sigma_s \lambda_s^2}{2} \right) ds \right].
\]  

(4.4)

In particular, if \( A_0 > kL_0 \), then the wealth process satisfies \( R_t \geq k \). If \( m = 1 \), then (4.3) amounts to a buy-and-hold strategy: the pension fund buys \( k \) shares of the vanishing-maturity bond that pays \( LT \) at date \( T \), and \( (1 - k/R_0) \) shares of the unconstrained portfolio.

4.2 Calibration of the Strategy

For arbitrary choices of the unconstrained strategy and of the multiplier, strategies of the form (4.3) are not utility-maximising. But they still have the attractive property that they allow for a protection of a minimum funding ratio \( k \), while opening access to the upside performance generated by the unconstrained strategy. This has interesting implications in integrated ALM. Indeed, a strategy that respects a minimum funding ratio protects the interests of the pensioners, and a strategy that outperforms the liability-hedging portfolio is likely to generate surpluses which will benefit equity holders. Hence a dynamic strategy of the form (4.3) combines protection of pensioners’ interests and access to performance for equity holders.

The upside performance of the CPPI strategies depends on two main factors. The first factor that will drive the performance of the CPPI strategy is the multiplier \( m \). Indeed, a higher \( m \) means a higher allocation to stocks, hence an extended access to upside. The second factor is the underlying unconstrained strategy, which must of course deliver substantial performance. If a strategy of the form (4.5) is to be adopted, the risk aversion must not be high. Indeed, a high \( \gamma \) would imply that the CPPI strategy would be almost entirely invested in the LHP, which would cancel the benefits of dynamic risk control. In contrast, taking \( \gamma = 1 \) implies that the unconstrained strategy is fully invested in the PSP, which generates a high performance. In what follows, we will in fact follow common practice by assuming that the unconstrained strategy is invested in

\[
w_t^* = \left( 1 - \frac{k}{R_t} \right) \left[ \lambda_{PSP} w_{tPSP} + \left( 1 - \frac{1}{\nu} \right) w_{tLHP} \right] + \frac{k}{R_t} w_{tLHP},
\]

where the PSP is the portfolio of bonds and stock that achieves the highest Sharpe ratio:

\[
w_{tPSP} = \frac{1}{1'/Q_t^{-1}(\rho')^{-1} \rho' \lambda} Q_t^{-1}(\rho')^{-1} \rho',
\]

and \( \sigma_{PSP} \) and \( \lambda_{PSP} \) are its volatility and Sharpe ratio, that is:

\[
\sigma_{PSP} = \sqrt{\lambda' \rho(\rho')^{-1} \rho' \lambda \left| 1'/Q_t^{-1}(\rho')^{-1} \rho' \lambda \right|},
\]

\[
\lambda_{PSP} = \sqrt{\lambda' \rho(\rho')^{-1} \rho' \lambda \times \text{sign} \left[ 1'/Q_t^{-1}(\rho')^{-1} \rho' \lambda \right]}.
\]

The proof of this result is reminded in appendix A.2.
4. CPPI Strategies in ALM

the stock only.\(^3\) Hence the CPPI weights become:

\[ w_t = m \left( 1 - \frac{k}{R_t} \right) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{k}{R_t} w_t^{LHP}, \quad (4.6) \]

Turning to the floor, we remark that the minimum level \(k\) in (4.3) can have two interpretations. In the first interpretation, which is the one discussed in MM2010b, it is regarded as an exogenous parameter, imposed by the regulator. The second interpretation, which we favour in this section, is that \(k\) is a technical risk management parameter chosen by the pension fund in order to parameterize its allocation. From the pensioners’ perspective, it would be ideal to take \(k\) at least equal to 100%, because this would guarantee that the pension fund can serve the promised payment. But as noted above, the initial funding ratio must be greater than \(k\), so a pension plan that is underfunded cannot credibly commit to having a final funding ratio greater than 100%. For any initial funding status, \(k\) can be set to any value, as long as it satisfies:

\[ 0 \leq k \leq R_0. \]

This constraint must be imposed in order to ensure that the risk budget in (4.3) is nonnegative. The pension fund must take into consideration two opposite effects. On the one hand, if \(k\) is set close to zero, then the risk-controlled strategy is equivalent to the unconstrained one, and nothing is gained or lost by taking the more sophisticated approach (4.3). In particular, the terminal funding ratio is just bounded from below by \(k\), and can thus be close to zero if the unconstrained strategy performs poorly. If \(k\) is set to the highest value \(R_0\), then the risk budget is zero and the funding ratio is constant. In the case the pension fund is initially underfunded, it makes little sense to take this option, because the probability of recovering from the underfunded status is then zero. If the pension fund is overfunded, this funding status is protected, but there is no access to upside.

4.3 Numerical Illustration

In a simple model with liabilities fixed in nominal terms and a constant interest rate, MM2010a show that a higher \(m\) translates into an increase of equity value, without decreasing the fair value of liabilities, provided the threshold \(k\) is set to 100% at least, and the initial funding ratio \(R_0\) is greater than \(k\). The purpose of the present numerical analysis is to extend this preliminary analysis to a richer setting with inflation-indexed liabilities and stochastic interest rates. We thus assume that liabilities are fully indexed on inflation.

4.3.1 Base case parameter set

Parameter values are set as indicated in table 1. The volatility of the short-term rate is set to 1.5% per year, which falls within the range of values usually assumed for this parameter (which are typically comprised between 1% and 2%). The speed of mean-reversion in the short-term rate was set to 20%: the value retained for this parameter exhibits much more variability from one paper to the other; a relatively high value was retained here in order to ensure that the probability of simulating negative rates stays small. The market price of interest rate risk varies considerably across papers.

\(^3\) This strategy is utility-maximising only if the risk aversion is 1 and parameter values are such that the PSP does not contain nominal or indexed bonds. If these conditions are not satisfied, it does not maximise expected utility, but it is still attractive for its risk-control properties and its potential upside performance.
### 4. CPPI Strategies in ALM

too. The value of 20%, together with the other interest rate parameter values, implies that the expected return over a 10-year bond over the cash is 1.30% per year. The instantaneous volatility of the stock index is set to 20%, which is close to the annualised volatility of a diversified stock index such as the S&P500 over a short holding period. The Sharpe ratio is set to 50%, which is slightly higher than the long-term value of 44.1% calibrated by Munk et al. (2004), but substantially higher than the historical Sharpe ratio of the S&P500. The expected inflation rate was set to 3.50% per year, and the volatility of unexpected inflation to 1.80%. These values are close to the values obtained by the authors through a direct calibration to the US Consumer Price Index over the period Q2 1961 to Q3 2011 (the calibrated values were 4.04% for the expected inflation rate and 1.63% for the volatility).

Most of the correlation parameters were set to zero, so as to express neutral views as to the correlations between the state variables. The correlation between the short-term rate and the price index was taken equal to 15% in order to reflect a positive correlation between the nominal short-term rate, which is set by monetary authorities, and realised inflation. The authors’ calibration to US market data on T-bills and CPI gave a value of 25.65%.

Finally, the correlation between the stock index S and the unlevered asset value of the firm was set to 50%: this positive value indicates that the firm covaries positively with the broad stock index. MM2010a discuss the impact of this parameter on the values of the claims for different allocation and pension funding policies.

Parameters other than market parameters were set to the same values as in MM2010a. The face values of promised payments to pensioners and bondholders were set to $50 each. This implies a total face value of $100, which is equal to the initial capital x made available to the firm and pension fund. The proportional bankruptcy costs in case of corporate default are set to 50%, and the corporate tax rate to 35%, as in Leland (1994).

#### 4.3.2 Protocol

We assume that the pension fund implements strategy (4.6) and that the portfolio is rebalanced at dates $t_0 = 0 < t_1 < \cdots < t_n$ evenly spaced of $\Delta t$. Between two consecutive dates, the portfolio is left buy-and-hold and evolves as:

$$A_{t_i+1} - A_{t_i} = A_{t_i}w_t^*(\frac{S_{t_i+1} - S_{t_i}}{S_{t_i}} \frac{\ln(1+r)}{r} + \frac{A_{t_i} - (1+r)A_{t_i}}{r} \frac{\ln(1+r)}{r})$$

$$+ A_{t_i} [1 - w_t^*] \frac{S_{t_i+1} - S_{t_i}}{S_{t_i}}.$$

It should be noted that because the risk budget is not monitored continuously, it is possible, at least in theory, that the wealth falls below the floor between two rebalancing dates. In particular, it may happen that the risk budget at some date $t_i$ is negative. Since a negative risk budget does not make sense and would imply a short position in the unconstrained strategy, we consider, at each date, the maximum value among the theoretical risk budget and zero. In case the theoretical risk budget is negative, it is replaced by the value 0 in (4.6), so the allocation to the PSP is zero and the portfolio is fully invested in the LHP. Hence the vector of weights at date $t_i$ is given by:

$$w_t^* = \max\left[ m \left( 1 - \frac{k_i}{A_{t_i}} \right) , 0 \right] w_t^*$$

$$+ \min\left( \frac{k_i}{A_{t_i}} , 1 \right) w_t^{LHP}.$$
4. CPPI Strategies in ALM

In subsequent numerical illustrations, we consider a monthly rebalancing frequency, which is commonly adopted in practical implementations. The payoff of the CPPI strategy was simulated 50,000 times, together with the minimal pricing kernel given in equation (2.5) and the terminal unlevered asset value $V_t$. Averaging the simulated discounted payoffs $M_t^P$, $M_t^E$ and $M_t^D$, we obtain estimates for the prices $P_0$, $E_0$ and $D_0$.

4.3.3 Results

Figures 1, 2 and 3 show the values of the claims for different values of the minimum funding level $k$, of the initial funding ratio $R_0$ and of the multiplier $m$. The solid lines represent the values of the claims when the pension fund implements the strategy (4.6) on a monthly basis, with a minimum $k$ equal to 100% or 80%. For comparison purposes, the dashed lines represent the values of the claims when the pension fund invests only in the LHP, and the dotted lines plot the values when it invests only in the stock index. By construction, these values are independent from $k$ and $m$.

On figure 1, the fair value of pension claims with the CPPI strategy is independent from the multiplier and from the initial funding ratio. This happens because the choice $k = 100\%$ together with an initial funding level greater than 100% ensures that the pension fund makes the promised payment at date $T$ in all states of the world. Because pensioners do not have access to surpluses, they are not affected in any way by the riskiness of the strategy chosen by the pension fund. Equity holders, on the other hand, benefit from a more risky investment policy, and equity value is increasing in the multiplier. Moreover, the conservative strategy that would consist of investing only in the LHP would lead to a lower equity value than the CPPI strategy, even for the lowest values of the multiplier. Of course, the
CPPI strategy provides only partial access to the upside performance of the stock index, so equity value is in general lower if the pension fund implements a CPPI than if it invests only in the stock. But the results show that it is possible to improve the value of equities by setting the multiplier to a higher value, without adversely impacting pensioners.

If the minimum funding level guaranteed is only 80%, as in figure 2, the impact of an increase in the multiplier is qualitatively the same for equity holders: equity value is still increasing in the multiplier. The situation is different for pensioners: the fair value of pension claims is no longer the same with the CPPI and the conservative strategies. Furthermore, it is decreasing in the multiplier $m$, which shows that...
pensioners prefer the pension fund to take the safest possible strategy if it chooses the minimum funding level to be less than 100%. This effect can be explained by the fact that the aggressiveness of the policy chosen by the pension fund impacts pensioners in an asymmetric way. A more risky strategy implies more dispersion in the terminal funding ratio, hence more severe shortfalls. It is equally true that it also implies higher surpluses, but pensioners do not have access to them, so they only perceive the negative effect. In other words, since the upside potential is limited to 100% of the promised value, while the downside potential can be as low as 0% – the net effect is negative. In order to aggregate the respective impacts of the multiplier on equities and pension claims, we have also represented the sum \( P_0 + E_0 \). Interestingly, the results show that if the multiplier is sufficiently high, the

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**Figure 2: Impact of the multiplier of the CPPI strategy with minimum \( k = 80\% \) and unconditional indexation on inflation.**

The solid lines represent the fair values of the claims held by pensioners \( (P_0) \), equity holders \( (E_0) \) and debtholders \( (D_0) \), as well as the total value \( (V_0) \), when the pension fund follows the CPPI strategy (4.7). \( m \) denotes the multiplier of the strategy, and the minimum terminal funding level \( k \) is set to 80%. The dashed lines represent the values of the claims when the pension fund invests only in the liability-hedging portfolio, and the dotted lines the values when it invests only in the stock index. Pension liabilities are unconditionally indexed on inflation, and the initial funding ratio of the pension fund, \( R_0 \), can take on three values. Other parameters are set to their base case values (see table 1).
positive impact on equity holders more than compensates the negative impact on bondholders. Indeed, for a multiplier greater than a critical value which is close to 3, the sum $P_0 + E_0$ is higher than with any lower multiplier, and is also higher than with a strategy that fully invests in the LHP or in the stock index. If the minimum funding level is set to an even lower value, such as 36% in figure 3, it appears that the dynamic strategy leads to a sum $P_0 + E_0$ higher than the strategy invested in the stock index only for all values of the multiplier.

Regarding corporate debt, the value of bonds appears to always be decreasing in the multiplier, to be lower with the CPPI strategy than with the conservative one, and to be higher than with a strategy fully invested in the stock index, except if the multiplier is set to a value above 2.

Figure 3: Impact of the multiplier of the CPPI strategy with minimum $k = 36\%$ and unconditional indexation on inflation. The solid lines represent the fair values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$), when the pension fund follows the CPPI strategy (4.7). $m$ denotes the multiplier of the strategy, and the minimum terminal funding level $k$ is set to 36%. The dashed lines represent the values of the claims when the pension fund invests only in the liability-hedging portfolio, and the dotted lines the values when it invests only in the stock index. Pension liabilities are unconditionally indexed on inflation, and the initial funding ratio of the pension fund, $R_0$, can take on three values. Other parameters are set to their base case values (see table 1).
The same observation was made in MM2010a when there was a nonnegative correlation between the stock index and the value of the firm. The situation is similar here because the parameter $\rho_{SV}$ has been set to 50%. A possible explanation is that debtholders hold an option written on the assets of the firm and the pension fund. This option has a rather complicated payoff (see (3.5)), but as a first approximation, it can be seen as a short position in a put written on the aggregate asset $A_T + V_T$, with a strike equal to $L_T + D$. No diversification takes place between the portfolio held by the pension fund and the assets of the firm here, because the pension fund is long the stock and the indexed bond, hence has a long exposure to stock risk and inflation risk, and a short exposure to interest rate risk. With zero correlations between $V$ and the short-term rate and between $V$ and inflation, and a positive correlation between $S$ and $V$, this results in a positive correlation between $A$ and $V$. When the multiplier increases, this correlation also increases, as does the volatility of $A + V$. Because the put has positive vega, the value of the put decreases, hence the value of corporate bonds decreases. Overall, the total value of the firm and pension fund appears to be less with the CPPI strategy than with the strategy fully invested in the LHP. No monotonic pattern emerges, however, in its variations with respect to $m$. We shall see in the next sections that a reduction in the cost of insurance implied by either imposing a maximum funding ratio, or by designing more complex strategies taking into account the health of the sponsor company, can actually result in a net positive effect of increases in risk taking.

In figure 4 we have repeated the above analysis but assuming that indexation on inflation was conditional. Through conditional indexation, pensioners have a partial access to the surpluses of the pension fund: they are promised a nominal payment $L$, but as soon as the real funding ratio exceeds 100%, they in fact receive $L$ plus inflation. It appears that the fair value of pension claims is no longer independent from the multiplier as it was when pensioners had no access to surpluses and the pension fund was initially fully funded (see figure 1). This dependence comes from the opened access to surpluses. Moreover, in contrast with figures 2 and 3, the value of pension claims is not necessarily decreasing in the multiplier. For instance, when the initial ratio is 110%, it is increasing from $m = 0$ to about $m = 2.5$, and then decreases again. Interestingly, this range coincides with the range of values of $m$ over which the probability of indexation increases. In conclusion, if the initial funding ratio is close to the minimum funding level, the probability for pensioners to receive an indexed payment and the fair value of pension claims reach a maximum for some value of the multiplier near 2.5. When the initial ratio is sufficiently high, then the probability of indexation is already close to 1 if the pension fund takes a safe strategy so, pensioners do not benefit from an increase in risk taking. It can also be noted that equity value is minimal precisely at the value of the multiplier that maximises pension claims.

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Figure 4: Impact of the multiplier of the CPPI strategy with minimum \( k = 100\% \) and conditional indexation on inflation. The solid lines represent the fair values of the claims held by pensioners \( (P_0^f) \), equity holders \( (E_0^f) \) and debtholders \( (D_0^f) \), as well as the total value \( (v_0^f) \), when the pension fund follows the CPPI strategy (4.7). \( m \) denotes the multiplier of the strategy, and the minimum terminal funding level \( k \) is set to 100%. The dashed lines represent the values of the claims when the pension fund invests only in the liability-hedging portfolio, and the dotted lines the values when it invests only in the stock index. Pension liabilities are conditionally indexed on inflation, and the initial funding ratio of the pension fund, \( R_0 \), can take on three values. Other parameters are set to their base case values (see table 1).
4.4 Imposing Leverage Constraints

With the adjustment for negative risk budgets in (4.6), the weights allocated to the stock and the bond in the CPPI strategy are always positive. However, since leverage has not been precluded ex-ante, their sum may exceed one, which implies a negative position in cash. The problem is all the more acute if the multiplier $m$ is high, because a high $m$ magnifies the effect of the risk budget, at least when it is not infinitesimal, and is thus likely to make the weight allocated to the stock index exceed one. Unless precluded from regulatory reasons, leverage would not be a concern per se if rebalancing was performed in continuous time. But in discrete time, the value of a leveraged portfolio may turn negative if the value of the risky asset strongly decreases between two rebalancing dates.

To see under which conditions leverage may or may not occur, we first write the sum of the weights allocated to the stock and the bond:

$$\max \left( m \left( 1 - \frac{k L_i}{A_i} \right) , 0 \right) + \min \left( \frac{k L_i}{A_i} , 1 \right).$$

Clearly, if the theoretical risk budget is nonnegative, the portfolio is unleveraged if, and only if, the value of $m$ is smaller than one, which is not of interest since we typically want to consider values for $m$ that are greater than one. In general, it is therefore impossible to exclude leveraged portfolios simply by controlling the parameter $m$, and leverage constraints must therefore be applied ex-post. Formally, we apply the following scaling rule to the weights of the CPPI strategy (4.6):

$$\tilde{w}_i^j = \frac{w_i^j}{1 + \left( w_i^{stock} + w_i^{ind. bond} - 1 \right)^+},$$

at each trading date $t_0, ..., t_n$, for $j$ denoting the stock and the zero-coupon bond. This means that the actual weights allocated to these two assets are divided by the sum of the theoretical weights given by the CPPI strategy if the sum exceeds one, and are left unchanged otherwise. As argued above, the sum of the weights is always greater than one if the multiplier is itself greater than one. As a consequence, the leverage constraints are always binding, so the position in cash after the constraints are applied is zero.

Figures 5 and 6 show the values of the claims when the pension fund implements a CPPI strategy and applies leverage constraints ex-post. We have focused on the values 100% and 80% for the minimum terminal funding level, and we have considered the same range of values for the multiplier and the same values for the initial funding ratio as in the leveraged case. From a qualitative perspective, it appears that introducing these constraints does not modify the effect of the multiplier on the values of the claims. Quantitatively, however, they tend to reduce this effect. For instance, with a minimum funding level of 100%, an initial funding ratio of 200% and no leverage constraints, equity value increases from about $55 to $62 when the multiplier increases from 1 to 5. With the constraints, the increase is from $55 to $57 only. This is because the benefits derived from implementing the CPPI strategy in terms of access to the upside are reduced by the presence of the
leverage constraints. A similar observation can be made for all claims and for other values of $k$ and $R_0$. The reduction in the impact of $m$ is even more pronounced when the initial ratio $R_0/k$ is high, because these are the situations where the risk budget is greater, and thus the leverage constraints are more binding.

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4.5 Relaxing the Minimum Funding Constraint in Exchange for Unconditional Indexation

The question of choosing the minimum funding level can be analysed in relation with the ongoing debate in the Netherlands on the change of regulation. Under the current regulatory regime, FTK1, liabilities are measured in nominal terms, and only conditional indexation on inflation is promised to pensioners. On
the other hand, the regulation imposes a “high” minimum funding requirement of 105% in an attempt to protect the interests of pensioners. The new regime, FTK2, is to set the focus on full inflation indexation and a real measure of liabilities, in exchange for a relaxation of minimum funding constraints. The exact details of this new regulatory framework are not entirely clear at this stage: in particular, the minimum funding level is not known to date, and the discount rate applied to liabilities is still to be specified.8

4.5.1 Protocol
A formal comparison of FTK1 and FTK2 is beyond the scope if this paper, if only because the exact details of the FKT2 regulation are yet to be finalised. Thus, we do not attempt to accurately model

Figure 6: Impact of the multiplier of the CPPI strategy with leverage constraints and a minimum k = 80%.
The solid lines represent the values of the claims held by pensioners (P₀), equity holders (E₀) and debtholders (D₀), as well as the total value (v₀). The pension fund implements a risk-controlled strategy of the form (4.7), and leverage constraints have been imposed ex-post. m denotes the multiplier of the strategy, and the minimum terminal funding level k is set to 80%. The dashed lines represent the values of the claims when the pension fund invests only in the liability-hedging portfolio, and the dotted lines the values when it invests only in the stock index. Pension liabilities are unconditionally indexed on inflation, and the initial funding ratio of the pension fund, R₀, can take on three values. Other parameters are set to their base case values (see table 1).
FTK1 and FTK2, and we instead perform a simple comparison of two hypothetical regulatory regimes:

1. Under regime 1 (conditional indexation), pensioners are promised a fixed nominal payment \( L \), but they receive \( L \Phi_T \) if the terminal wealth of the pension fund satisfies \( A_T \geq L \Phi_T \);

2. Under regime 2 (full indexation), pensioners are promised a fixed real payment \( L \), that amounts to a nominal payment \( L \Phi_T \).

Under regime 1, the minimum regulatory funding level is set to 105%, which reflects the current MFR of FTK1. Regime 1 is a stylised version of FTK1, while regime 2 represents a reasonable approximation of what the future FTK2 could be. The objective of the numerical illustration is to assess the impact of a decrease in this minimum funding level under regime 2, to analyse the trade-off in terms of offering full indexation versus lowering the minimum funding ratio requirement.

It should be noted that the funding ratio has two different definitions under regimes 1 and 2. Since the promised payment under regime 1 is fixed in nominal terms, the funding ratio is computed as:

\[
R_{0}^{\text{nom}} = \frac{A_0}{LB(0, T)}.
\]

Under regime 2, the promised payment is fixed in real terms, hence the funding ratio is computed as:

\[
R_{0}^{\text{real}} = \frac{A_0}{L(0, T)}.
\]

For a given face value \( L \) of the promised payment to pensioners and for a given initial endowment \( A_0 \) to the pension fund, we have: \( R_{0}^{\text{real}} < R_{0}^{\text{nom}} \), because the indexed bond is more expensive than the nominal one.

### 4.5.2 Numerical Results

On figure 7, the initial funding ratio \( R_{0}^{\text{nom}} \) is set to 116%, 158% or 210%. These values imply that the ratio \( R_{0}^{\text{nom}}/k \) is equal to 110%, 150% or 200%, as assumed in figures 1 to 2. Keeping \( A_0 \) and \( L \) constant, we compute the values of the claims under regime 2 for different values of the minimum regulatory funding level \( k \). Because of the constraint (4.2), this level must be less than or equal to \( \Phi_T \).

The values of the claims under regime 2 are represented by the solid lines, and the values under regime 1 are represented by dots. Unsurprisingly, the value of pension claims is higher under regime 2. It can be explained by the fact that pensioners are promised a higher payoff in all states of the world, due to the unconditional indexation on inflation. The value of pension claims is also increasing in the minimum funding level \( k \), up to \( k = 100\% \). Indeed, when the minimum is 100%, they receive the promised payment in all states of the world, so the value of their claims is simply equal to the price of a default-free indexed bond that pays \( L \Phi_T \). If the minimum is less than 100%, there is a positive probability of deficit at date \( T \), and the average size of this deficit increases when \( k \) decreases, which in turn decreases the fair value of the claims.

With most parameter values, the value of corporate bonds appears to be less under regime 2 than under regime 1, that is to say that credit spreads are higher under regime 2. Again, this comes from the fact
that the promised payment to pensioners is higher if there is unconditional indexation on inflation. The only case where bondholders will benefit from the new regime is when the pension fund is very well funded (with a funding ratio under regime 2 of approximately 148%) and the minimum funding level under regime 2 is sufficiently high (namely greater than 100%). More generally, bondholders benefit from a higher k because it decreases the likelihood of a severe underfunding of the pension plan: if such situations are avoided, then the sponsor is less likely to be called to contribute, which allows it to keep money available for debt redemption.

For all parameter values, equities have a lower value under regime 2. This effect still relies on the unconditional indexation of promised pension payments on inflation.

Figure 7: Impact of decreasing minimum funding requirements in exchange for unconditional indexation with constant face value L. The circles represent the values of the claims under a regime where promised liabilities are fixed in nominal terms and conditionally indexed on inflation, and the minimum funding level is 105% (regime 1). The solid lines represent the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$), under a regime where liabilities are unconditionally indexed on inflation, and the pension fund is subject to a minimum funding requirement $k$ (regime 2). Under both regimes, the pension fund implements the CPPI strategy (4.7) with a multiplier equal to 3. The initial funding ratio under regime 1 is denoted with $R_{0}^{f}$, and the initial funding ratio under regime 2 is denoted with $R_{0}^{f_{2}}$. Other parameters are set to their base case values (see table 1).
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A similar result was reported in MM2010a using fixed-mix strategies. It can also be seen that a higher $k$ decreases equity value. The origin of this effect is in the decrease in the volatility of the wealth generated by the CPPI strategy. Indeed, a higher $k$ implies a lower allocation to the risky block in (4.5), which tends to reduce the variability of the wealth of the pension fund. Because equity holders hold a call written on this wealth through their access to surpluses, they are hurt by this decrease in the volatility. Considering the overall effect of the change of regulation, it can be seen that the total value of the firm and pension fund is in most cases lower under regime 2. This comes as no surprise, given that out of the three groups of claim holders, only one benefits from this change.

It can be argued that the previous comparison is not fair, because it assumes that the real promised payment to pensioners under regime 2 equals the nominal promised payment of regime 1. In practice, the sponsor might be tempted to decrease the face value of the promised payment in regime 2, so as to counterbalance the effect of unconditional indexation. To illustrate what impact would then be observed on the claims, we perform a second comparison between the two regimes. Assuming that the nominal promised payment to pensioners under regime 1 is $L = 50$, we decrease $L$ under regime 2 until the fair value of pension claims reaches the same value as under regime 1. Panel (a) in figure 8 compares the face values in both regimes. The face value in regime 2 that makes pensioners indifferent between the two settings appears to be decreasing in the minimum funding level. This happens because other things being equal, the fair value of pension claims is increasing in this parameter, as evidenced by figure 7. It can also be noted that the decrease in face value is far from insignificant: for instance, if the minimum funding level under regime 2 were as high as under regime 1, the sponsor would have to decrease the face value from $50$ to $36$ in order to leave the fair value of pension claims unchanged. This shows that indexation on inflation always benefits to pensioners if it is not compensated by a decrease in the real promised payment. Panel (b) shows the impact of the change in regulatory framework on the fair values of the claims. It can be seen that equity value is now higher under regime 2. The qualitative impact of an increase in the minimum funding level $k$ on the values of the claims is the same here as in the previous comparison. As a conclusion, if the face value of promised payment to pensioners is the same in regime 2 as in regime 1, then only pensioners will benefit from the change of regulatory regime. In particular, the change of regulatory framework will be costly for equity holders. In order to cancel this cost, it is necessary to decrease the face value of pension liabilities.
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Figure 8: Impact of decreasing minimum funding requirements in exchange for unconditional indexation with decreasing face value $L$. In regime 1, pension liabilities are fixed in nominal terms and conditionally indexed on inflation, and the minimum funding level is 105%. In regime 2, liabilities are unconditionally indexed and the minimum funding level is $k < 105\%$. The face value of liabilities in regime 1 is $50$, and the face value of pension liabilities is a constant $L$ such that the fair value of pension liabilities is the same under both regimes. Panel (a) shows the face value of liabilities in regime 1 (circle) and in regime 2 (solid lines). Panel (b) shows the fair value of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$). Under both regimes, the pension fund implements the CPPI strategy (4.7) with a multiplier equal to 3. The initial funding ratio under regime 1 is denoted with $R_0^{\text{opt}}$, and the initial funding ratio under regime 2 is denoted with $R_0^{\text{bal}}$. Other parameters are set to their base case values (see table 1).

(a) Face value of liabilities.

(b) Values of claims.
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In this section, we present several variants of the basic CPPI strategies analysed in the previous section. First, we measure the opportunity cost of using nominal bonds, as opposed to real bonds, in the LHP when liabilities are indexed on inflation. Then, we impose a cap on the terminal funding ratio, which will be shown to allow for a decrease in the cost of insurance. Finally, we discuss the protection of a floor linked to the regulatory liability value using a corporate bond in the LHP. These variants are analysed independently from each other.

5.1 Opportunity Cost of Imperfect Inflation Hedging

The strategy (4.5) requires the use of an inflation-indexed bond, both in the PSP and in the LHP. In the presence of possible capacity constraints in the inflation-linked bond market, one might be tempted to use a nominal bond in place of the real bond. In this subsection, we thus consider a strategy of the form:

\[ w_t = m \left( 1 - \frac{kL_t}{A_t} \right) w^\mu_t + \frac{kL_t}{A_t} \beta_t w^{LHP}_t, \quad (5.1) \]

where \( w^{LHP}_t \) is an imperfect LHP invested in the nominal bond only:

\[ w^{LHP}_t = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \]

and \( \beta_t \) is the beta of the indexed zero-coupon maturing at date \( T \) with respect to the nominal bond:

\[ \beta_t = \frac{\sigma'(T-t)' \sigma(\beta)(T-t)}{\|\beta(T-t)\|^2} = 1 - \frac{\sigma(\Phi)\beta(\Phi)}{\sigma'(T-t)} . \]

It should be noted that the imperfect LHP in (5.1) is multiplied by a beta. The use of the beta can be rationalised in a utility maximisation framework: if the agent maximises the expected utility from a terminal funding ratio, then the optimal strategy involves a long position in the portfolio \( \beta^{LHP} w^{LHP} \), where \( w^{LHP} \) is the portfolio that maximises the instantaneous squared correlation with the liability process, and \( \beta^{LHP} \) is the beta of liabilities with respect to the LHP (see Amenc et al. 2010). In the expression of the CPPI strategy (4.3), no beta is apparent because the beta is precisely equal to 1. More generally, if one replaces the true LHP by an imperfect LHP, that is, by a portfolio that does not maximise the squared correlation with liabilities, it makes still sense to multiply the imperfect LHP by the beta. This multiplication will ensure that the allocation to the imperfect LHP is increasing in the correlation between LHP and liabilities: in particular, if the correlation is zero, then the LHP is simply regarded as useless for the purpose of hedging liabilities. Moreover, the beta is decreasing in the volatility of the LHP, which guarantees that the allocation to the LHP is decreasing in this volatility: this property is natural given the preferences of investors for low volatility.

To illustrate the properties of (5.1) in an integrated ALM context, we assume that the pension fund implements the strategy on a monthly basis. Figure 9 compares the distribution of the terminal funding ratio achieved with a perfect hedge of inflation to that obtained by using a nominal zero-coupon bond in the LHP. As expected, the strategy that uses an imperfect LHP is unable to guarantee that the terminal funding ratio is at least equal to the minimum level \( k \), taken here equal to 100%. The probability of a shortfall, however, is rapidly decreasing in the initial funding ratio. If the pension fund starts from a funding ratio of 105%, the probability is
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only 14.3%, and it falls to 4.39% if the initial ratio is 120%. This result makes intuitive sense: if the pension fund is very well funded at the outset, then it benefits from a substantial "margin for error" that can partly absorb the losses incurred by the imperfect replication of liabilities.9

Figure 9 also reports the values of the claims with both LHPs. The value of pension claims slightly decreases when liabilities are imperfectly hedged with a nominal bond. This decrease takes place because the pension fund is more often in situations where it cannot make the promised payment. But as noted above, the shortfall probabilities only slightly increase. Moreover, the histograms show that the shortfalls are of limited size. As a consequence, the decrease in the fair value of pension claims is very modest – it never exceeds 0.5%. More surprisingly, the value of equities increases slightly. This can be explained as follows. As a first approximation, equity holders have a long position in a call option written on the assets of the firm and the pension fund, with a strike price equal to the sum of promised payments to pensioners and debtholders. When liabilities are imperfectly hedged, the volatility of the ratio $\frac{A_T}{L_T}$ increases, as is clear from the histograms, which leads to an increase in the volatility of the underlying of the call option. But the magnitude of the effect is extremely small – the increase is at most 0.79%. Finally, imperfect hedging of inflation risk within liabilities has a small positive impact on the total value of the firm and pension fund. This may be due to an increase in the tax shield, because the sponsor is more likely to have to make an additional contribution at the terminal date when inflation risk is not fully hedged. Of course, if the required contribution is too large, the sponsor defaults, which generates bankruptcy costs. Such high contributions are, however, unlikely to be necessary, because the shortfall is never large. Hence, the increase in the tax shield more than compensates the increase in bankruptcy costs, and the final effect on the total value is positive. As for the individual claims, the magnitude is very small.

In conclusion, the imperfect hedging of inflation risk does not appear to have a prohibitive cost, at least for our choice of parameter values. Due to the low volatility of realised inflation, the distribution of the terminal funding ratio of the pension fund is hardly affected, and the value of pension claims decreases by a small amount. The increase in the volatility of the funding ratio can even have positive effects for equity holders, who hold a call written on the assets of the firm and the pension fund. These effects would likely be magnified in an economic regime with higher inflation uncertainty.

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9 - It must be noted that these results of course depend on parametric assumptions. Comparing the dynamics of the nominal bond and of the indexed one in our model (see (2.1) and (2.2)), it can be seen that they differ only by the short-term volatility of the CPI, $\sigma_{\Phi}$, and the price of inflation risk, $\lambda_{\Phi}$. With our choice of parameter values, $\lambda_{\Phi}$ is zero, so both bonds have the same expected excess return. They do not have the same volatility because $\sigma_{\Phi}$ is not zero, but the difference is small, because $\sigma_{\Phi}$ is only 1.80%.
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Figure 9: Impact of imperfect inflation hedging.

These figures show the distribution of the terminal funding ratio, $R_T$, when the pension fund follows a CPPI strategy. The safe asset of the CPPI is either an indexed bond (left column) or a nominal bond (right column), which leads respectively to dynamic strategies (4.7) and (5.1). Also reported are the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$), and the shortfall probability of the pension fund. The minimum terminal funding ratio, identified by the vertical dashed line, is 100%, and the multiplier is set to 3. Pension liabilities are unconditionally indexed on inflation, and three values of the initial funding ratio, $R_0$, are considered: 105%, 110% and 120%. Other parameters are set to their base case values (see table 1).

(a) Initial funding ratio $R_0 = 105\%$.

(b) Initial funding ratio $R_0 = 110\%$.

(c) Initial funding ratio $R_0 = 120\%$. 
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5.2 Imposing A Cap on the Funding Ratio

Insurance against downside risk has an (implicit) opportunity cost because it decreases the access to the upside performance of the unconstrained strategy. As shown by MM2010b, the introduction of a maximum funding ratio constraint allows for a decrease of the cost of insurance against downside risk, by giving up the access to the upside potential of risky assets beyond exceedingly large funding ratio levels. In this section we extend the basic CPPI strategies in order to incorporate a maximum target funding ratio. The addition of a cap on the terminal funding ratio can be done by explicitly requiring that the terminal funding ratio satisfies $R_T \leq k'$. In this subsection, we consider a heuristic strategy that implicitly takes into account the maximum funding constraint:\(^{10}\)

$$w_t = m \left[ \left( 1 - \frac{k}{R_t} \right) I_{\{k \leq R_t \leq \frac{k+k'}{2} \}} + \left( \frac{k'}{R_t} - 1 \right) I_{\{\frac{k+k'}{2} < R_t \leq k' \}} \right] w_t^a + \left[ \left( 1 - \frac{k}{R_t} \right) I_{\{k \leq R_t \leq \frac{k+k'}{2} \}} - \left( \frac{k'}{R_t} - 1 \right) I_{\{\frac{k+k'}{2} < R_t \leq k' \}} \right] w_t^{LHP},$$

(5.2)

where $k'$ is a maximum funding ratio. Because of the indicator function in (5.2), one might be concerned about the continuity of the weight as a function of $R_t$. In fact, the vector is continuous, as can be shown by checking that the left and right limits of $w_t$ at the midpoint are equal:

$$\lim_{R_t \uparrow \frac{k+k'}{2}} w_t = \lim_{R_t \downarrow \frac{k+k'}{2}} w_t = m \frac{k' - k}{k + k'} w_t^a + \frac{2k}{k + k'} w_t^{LHP}.$$  

The intuitive meaning of (5.2) is that when the current funding ratio, $R_t$, approaches the floor $k$ or the cap $k'$, the portfolio is entirely invested in the LHP so as to allow for smooth landing on both the floor and the cap values. The allocation to the unconstrained strategy is maximal when $R_t$ is close to the midpoint $\frac{k+k'}{2}$. The limits of $w_t$ at extreme points $k$ and $k'$ are thus the same as those of the explicitly constrained allocation.\(^{11}\) It is also worth noting that in contrast with the CPPI strategy with no cap, which is a buy-and-hold when the multiplier is set to 1, (5.2) is still a dynamic strategy, even when the multiplier is equal to unity. Hence, taking $m$ equal to 1 in the following numerical illustration does not mean that the strategy becomes static.

Figures 10 and 11 show the distribution of the terminal funding ratio when a cap is imposed and the multiplier is set to 1 or 3. A first comment is that the strategy (5.2) is effective in ensuring that the maximum funding constraint is satisfied at the terminal date. The interesting feature is that the conditional mean of the funding ratio, given that it falls between the minimum $R_t$ and the maximum $R_t$, is higher when the maximum constraint is imposed. This shows that if the pension fund is ready to give up funding levels that correspond to extremely good performance of the unconstrained strategy, it can (on average) expect higher funding levels if the unconstrained strategy delivers a moderate performance. Second, when the multiplier of the CPPI strategy is set to 3, the distribution of the constrained funding ratio is almost bimodal: most of the mass is concentrated near the minimum or near the maximum funding levels. As a higher multiplier
implies a higher allocation to the stock, the constraints are more often binding. A third observation is that the fair value of pension claims increases when the cap is imposed and the minimum funding level is less than 100%. This is because pensioners, who do not have access to the surplus, do not symmetrically treat over-funded and under-funded situations. In other words, there is no cost for them to implement strategies that lead to giving up access to the upside potential beyond a given threshold, while they benefit from the decrease in the cost of downside protection. Besides, we note that the value of corporate bonds increases in all cases, and equity value is decreasing.

In figures 12 and 13, we conduct a systematic analysis of the impact of the maximum funding level $k'$ on the values of the claims. First, we confirm that pensioners are insensitive to the choice of the cap when the minimum funding level $k$ is set to a value greater than 100%, as in figure 12. Indeed, they receive the promised payment...
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in all states of the world, and they do not have access to surpluses so their payoff will be the same whether or not the terminal funding ratio is capped to a maximum. In contrast, they benefit from the introduction of a cap if the initial funding level is less than 100%, as in figure 13. Moreover, we find that the highest benefits are to be expected when the cap is low. This can be attributed to the increase in the conditional mean of the funding ratio that was evidenced by figure 10. Indeed, by giving up funding levels that are in excess of $k'$, the pension fund enjoys higher funding ratios when the unconstrained strategy has moderate performance. Such a situation is relevant to pensioners, who hope that the pension fund will end up with a funding ratio of 100%, but have no particular interest for high funding ratios. The same mechanism is at work for bondholders, whether the minimum funding level is 100% or lower – the fair value of corporate bonds appears to be decreasing in $k'$, which means that it increases when the maximum funding constraint is made tighter. Indeed, they

Figure 11: Impact of maximum funding constraint on the distribution of the funding ratio when the multiplier equals 3.
This figure shows the distribution of the terminal funding ratio, $R_T$, when the pension fund follows the CPPI strategy with maximum funding target (5.2). Also reported are the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$) and the conditional mean of the funding ratio given that the funding ratio ends up between $k$ and $k'$. Pension liabilities are unconditionally indexed on inflation, and the pension fund implements a risk-controlled strategy of the form (5.2), and the multiplier of the strategy is set to 3. Other parameters are set to their base case values (see table 1).

(a) Minimum $k = 100\%$, maximum $k' = 120\%$, initial ratio $R_0 = 110\%$.

(b) Minimum $k = 70\%$, maximum $k' = 120\%$, initial ratio $R_0 = 100\%$. 
have an indirect access to pension fund surpluses, through the claim that equity holders hold on to these surpluses, but this access is limited. In the case of the sponsor becoming insolvent, but the pension fund enjoying surpluses, bondholders receive the proceeds of liquidation, \((1 - \alpha)V_T\), plus the surplus \(A_T - L_T\), but only to the extent that \(A_T - L_T\) does not exceed the sponsor’s deficit, \(D - V_T\). As a consequence, bondholders are only interested in moderate surpluses, and they thus benefit from a decrease in the cap value. The situation is of course different for equity holders: the figure shows that equity value is increasing in \(k’\), which means that equity holders prefer a high cap – a preference which can be explained by the fact that they have an unbounded access to surpluses. Moreover, they receive a fraction of these surpluses only if they are sufficiently large. As clearly shown in (3.4), the surplus must be at least equal to \(D - V_T\).

Figure 12: Impact of maximum funding constraint on claimholders with minimum funding level \(k = 100\%\). The solid lines represent the values of the claims held by pensioners \(P_0\), equity holders \(E_0\) and debtholders \(D_0\), as well as the total value \(v_0\). Pension liabilities are unconditionally indexed on inflation, and the pension fund implements a CPPI strategy of the form (5.2) with a multiplier equal to 1 and a minimum funding level equal to 100\%. The dashed lines represent the values of the claims when the pension fund invests only in the liability-hedging portfolio, and the initial funding ratio of the pension fund, \(R_0\), can take on three values. Parameters are set to their base case values (see table 1).
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Because of this constraint, equity holders are only interested in “large” surpluses. These are precisely the situations that are precluded when a cap is set on the funding ratio, which explains why $E_0$ is increasing in the cap value.

5.3 Control of the Regulatory Floor Value

In practice, pension funds are often subject to short-term regulatory constraints that impose a minimum value for the regulatory funding ratio at all dates:

$$R_{t}^{\text{req}} \geq k, \quad t \leq T. \quad (5.3)$$

Because the regulatory value of liabilities is always lower than the present value of the promised payment due to the credit spread.

![Figure 13: Impact of maximum funding constraint on claimholders with minimum funding level $k = 80\%$. The solid lines represent the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$). Pension liabilities are unconditionally indexed on inflation, and the pension fund implements a CPPI strategy of the form (5.2) with a multiplier equal to 1 and a minimum funding level equal to 100%. The dashed lines represent the values of the claims when the pension fund invests only in the liability-hedging portfolio, and the initial funding ratio of the pension fund, $R_0$, can take on three values. Other parameters are set to their base case values (see Table 1).](image-url)
the regulatory funding ratio $R_t^{eq}$ is higher than the ratio $R_t$. Hence, to comply with the regulation, it is sufficient for the pension fund to respect the constraint $R_t \geq k$ at all dates. As previously explained, this can be done by implementing a CPPI strategy such as (4.3), where the performance block can be any strategy, or (4.6), where the performance block is invested in stocks only. In both situations, the dollar amount allocated to this block is $A_t - kL_t$, a quantity that is lower than the risk budget that would be computed with respect to the regulatory floor, which would be $A_t - kL_t^{req}$.

Taking a greater risk budget would increase the access to upside of the unconstrained strategy, and we may therefore be tempted to consider the following risk-controlled investing (RCI) strategy, which is a modification to (4.7):

$$w_t = \max \left[ m \left( 1 - \frac{k}{R_t^{eq}} \right), 0 \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \min \left[ \frac{k}{R_t^{eq}}, 1 \right] w_t^{LHP}.$$

It should be noted that the LHP replicates the process $(L_t)$. In particular, the return over the period $[t, T]$ on a strategy fully invested in the LHP is $\log(L_T/L_t)$. On the other hand, the return on the regulatory liability value over the same period is $\log(L_T^{eq}/L_t^{eq})$. Hence the strategy fully invested in the LHP underperforms the regulatory value. As a consequence, there is no guarantee that strategy (5.4) will respect the regulatory constraint (5.3). In figure 14, we implement strategy (5.4) assuming that the minimum funding level is 100% and the LHP is invested in a default-free bond. Two values of the regulatory spread are considered: 100 basis points, which is our base case, and 200 basis points. Given that the default intensity under $\mathbb{P}$ is assumed equal to half the spread (see subsection 2.2), these values of the spread imply probabilities of default over 10 years of 4.88% and 9.52%, respectively. Some 50,000 paths for the strategy (5.4) were simulated, assuming a monthly rebalancing. The figure shows the probability of not respecting the regulatory constraint, for each horizon comprised between 0 and 10 years. It also reports the probability of not respecting the constraint $R_t \geq k$, which is higher than the probability of not respecting the regulatory constraint $R_t^{eq} \geq k$ since $R_t \geq R_t^{eq}$. At the terminal date, both constraints are equivalent, because the adjustment for the credit spread in the value of liabilities is zero: hence the two probabilities are equal at this date. The violations of the constraint $R_t \geq k$ come from the definition of the risk budget: if the risk budget was defined as in the basic CPPI strategy, where it is equal to $A_t - kL_t$, the constraint $R_t \geq k$ would be satisfied at all dates. The probability of violating the risk budget is far from negligible: it is equal to 17.10% at the terminal date when the spread is 100 bps. If the spread increases, the probability grows even larger, because of the increased difference between the values $L_t$ and $L_t^{eq}$. The violations of the regulatory constraint $R_t^{eq} \geq k$ have a different origin: they arise from the difference in the returns on the LHP and on the regulatory value of liabilities. The probability of not respecting this constraint is close to zero for horizons between 0 and 3 years, but it is then increasing in the horizon.

In order to have a LHP with returns that would be perfectly correlated to the returns on the liability portfolio under the...
5. Introducing More Complex CPPI Strategies

Figure 14: Probability of violating minimum funding constraints with CPPI strategy when the LHP is invested in a default-free bond. This figure shows the probability of violating minimum funding ratio constraints \( A_t \geq kL_t \) and \( A_t - kL_t^{mg} \) after \( t \) years, when the pension fund implements the CPPI strategy (5.4), and uses a default-free bond in the LHP. Pension liabilities are unconditionally indexed on inflation, and are discounted at the risk-free rate plus a regulatory spread equal to 100 bp or 200 bp, the minimum funding level \( k \), represented by the vertical dashed line, is 80%, the multiplier is 3, and the initial funding ratio is 88%. Other parameters are set to their base case values (see table 1).

(a) Regulatory spread \( s^{mg} = 100 \) bp.

(b) Regulatory spread \( s^{mg} = 200 \) bp.

regulatory valuation rule, one would need to invest in a security with price \( L^{reg} \). One can consider a defaultable bond that has a credit spread equal to \( s^{reg} \) as a proxy for the best hedging instrument with respect to changes in the regulatory value of the liabilities. Such a bond is modelled as in subsection 2.2, with a default intensity under \( Q \) equal to the credit spread. The price of the defaultable bond at date \( t \) is thus given by:

\[
X_t = \mathbb{I}_{\{t \geq \Theta\}} e^{-s^{reg}(T-t)} f(t, T).
\]

This bond is then used in the LHP of strategy (5.4). When it defaults, at the random date \( \Theta \), its value falls to zero, hence the wealth of the pension fund suddenly drops.13 In this case, we assume that the entire portfolio is reinvested in a default-free indexed bond that matures at date \( T \).

Figure 15 shows the probability of not respecting the constraints \( R_T \geq k \) and \( R_T^{mg} \geq k \) when the pension fund implements the CPPI strategy (5.4), but uses a corporate bond in the LHP. A first observation is that

13 - The drop would be more limited in size if the bond had a non-zero recovery rate.
the probability of respecting the regulatory constraint is significantly higher than when a sovereign bond was used. For instance, with a spread equal to 100 bps, the probability of violation at the terminal date falls from 17.10% to 3.89%. For horizons shorter than 3 years, the probability of not respecting the short-term constraint is almost equal to the probability that the issuer of the corporate bond has defaulted. Indeed, if the corporate bond defaults, then the value of the portfolio jumps downwards, hence it is unlikely that the pension fund will still be able to satisfy the minimum regulatory funding requirement. Conversely, as long as the issuer does not default, the corporate bond delivers higher performance than the default-free one. Indeed, the expected return under \( \mathbb{P} \) on the corporate bond before default \((t < \Theta)\) is given by the following formula (see Lando (2004)):

\[
\mu_t^X = r_t - D(T - t)\sigma^X \lambda + \sigma^X \Phi^X + v^P - v^Q
\]

\[
= r_t - D(T - t)\sigma^X \lambda^X + \sigma^X \Phi^X + \frac{1}{2} s^{reg}.
\]

The first two terms represent the expected return on a default-free indexed bond with the same maturity date as the corporate bond, but the last term represents an additional reward for default risk.

In figure 16, we examine whether the use of a corporate bond instead of a default-free bond in the LHP of strategy (5.4) has a positive impact on the values of the claims. The figure also performs a comparison of this strategy with the basic CPPI (4.7). As discussed above, strategy (5.4), whether it is implemented with a sovereign or with a corporate bond, yields a non-zero probability of violating the regulatory constraint at the terminal date. In particular, the deficit of the pension plan can be larger than if the basic CPPI, less aggressive, strategy is followed. As a consequence, the fair value of pension claims decreases: it is $46.23 with the standard CPPI, and it falls respectively to $45.73 and $45.42 when strategy (5.4) is implemented with a default-free bond or a corporate bond. The value of bonds issued by the sponsor company is also lower with the new strategy than with the basic CPPI: this effect is also due to an increase in the likelihood of large deficits, because in case of severe shortfall, the pension fund must call for a contribution of the sponsor, which puts the repayment of corporate debt at risk. The credit spread of the sponsor decreases if a corporate bond replaces the default-free bond in the LHP, but it stays above the value it would reach with the basic CPPI. The decrease in the credit spread can be explained by the decrease in the probability of shortfall, that was evidenced by figure 15. Investing in the corporate bond, the pension fund is more likely to enjoy a surplus at the terminal date, although it is also more likely to experience a large deficit. Bondholders indirectly benefit from surpluses, which help the sponsor to redeem the debt. For the same reason, the value of equities increases when the LHP in strategy (4.7) contains a corporate bond: it grows substantially, from $40.56 to $42.56. On the other hand, the value of equities is always higher when the floor is based on the regulatory value of liabilities than when it is based on the present value of the promised payment. This effect is due to the increase in the risk budget, which results in enhanced access to the upside of the stock. Overall, the total value of the firm is greater with the strategy (4.7) than with the basic CPPI strategy, and the largest increase is to be expected if a corporate bond is used in the LHP.
5. Introducing More Complex CPPI Strategies

At this stage, it is useful to note that one important limit in this analysis is the assumption that the default date of the traded corporate bond is independent from the financial market processes \( r, S \) and \( \Phi \), and from the unlevered asset value of the sponsor, \( V \). In particular, one might argue that, to the extent it can do so, the pension fund should select a corporate bond that is less likely to default when \( V_T \) is low or \( \Phi_T \) is high. Indeed, the states of the world where promised pensions are high and the sponsor is in bad financial condition are also those where the pension fund critically needs a high funding ratio. Such a high ratio is easier to reach if the corporate bond included in the LHP has not defaulted. Introducing some dependence between the random default date \( \Theta \) and other processes requires relaxing the assumption of a constant intensity of default. This extension poses technical challenges, and is left for further research.

Figure 15: Probability of violating minimum funding constraints with CPPI strategy when the LHP is invested in a corporate bond. This figure shows the probability of violating minimum funding ratio constraints \( A_t \geq k L_t \) and \( A_t \geq k L_t^{\text{reg}} \) after \( t \) years, when the pension fund implements the CPPI strategy (5.4) with a minimum funding level \( k \) is 80% and a multiplier equal to 3, and uses a corporate bond in the LHP. Also reported is the cumulated default probability of the corporate bond. Pension liabilities are unconditionally indexed on inflation, and are discounted at the risk-free rate plus a regulatory spread equal to 100 bp or 200 bp. The credit spread of the corporate bond is equal to the regulatory spread, and the initial funding ratio \( R_0 \) is 88%. Other parameters are set to their base case values (see table 1).

(a) Regulatory spread \( s_{\text{reg}} = 100 \) bp.

(b) Regulatory spread \( s_{\text{reg}} = 200 \) bp.
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Figure 16: Distribution of terminal funding ratio with a CPPI strategy aiming at protecting a minimum regulatory funding level. This figure shows the distribution of the terminal funding ratio when the pension fund implements the CPPI strategy (5.4) with a LHP invested in a defaultable indexed corporate bond, a minimum funding level \( k \) equal to 80% and a multiplier equal to 3. Pension liabilities are unconditionally indexed on inflation, and are discounted at the risk-free rate plus a regulatory spread equal to 100 bp. The credit spread of the corporate bond is equal to the regulatory spread, and the initial funding ratio \( R_0 \) is 88%. Other parameters are set to their base case values (see table 1).

(a) LHP invested in a default-free bond.

(b) LHP invested in a corporate bond.
6. Insuring Against Sponsor Risk
6. Insuring Against Sponsor Risk

As shown in section 5, introducing maximum funding ratio targets is an effective way to decrease the opportunity cost of downside risk protection for corporate pension plans. Another approach for reducing the opportunity costs consists of remarking that violations of the minimum terminal funding ratio can be tolerated, as long as the sponsor can make up for the deficit through extra contributions if and when needed. Indeed, the sponsor is constrained by the size of the assets available at date $T$, which is measured by $V_T - D$. The objective here is to design an integrated dynamic LDI strategy that would offer downside protection for pension assets only in those states of the world where the sponsor company is weak, as opposed to providing protection against downside risk of equity markets relative to the liabilities irrespective of whether or not the sponsor company can afford to make up for the deficit if it arises.

6.1 Trading Sponsor Risk

In subsection 6.2 below, we will assume that the pension fund has access to a security, or portfolio strategy, that perfectly replicates the payoff $V_T$. This implies, by absence of arbitrage opportunities, that a portfolio strategy exists, whose value at date $t$ is always $V_t$. We will refer to this strategy as a firm-hedging security. In practice, one would typically be tempted to use the stock of the firm to hedge against unexpected changes in the firm value. However, the value of equities can be written as an option on this underlying process. The return on the company stock, however, will not be perfectly correlated with the return on the firm value because the present value at date $t$ of the payoff $E_T$ defined in (3.4) is a function of time and of the current unlevered value $V_0$, but also a function of the current values of the short-term rate, $r_n$, the stock index $S_n$, and the price index $\Phi_t$. Because the value of equities can be regarded as a multiple asset option, impacted by multiple sources of risk, it cannot be perfectly correlated with the process $V$. Hence, to attain a perfect correlation with $V$, it is necessary to use a dynamic strategy that dynamically mixes different securities in order to cancel the impacts of interest rate risk, inflation risk and stock index price risk. While feasible in principle, implementing this trading strategy in practice would not be straightforward, and we will discuss below an incomplete market setting where an imperfectly correlated asset, which can be the stock of the firm or stock issued by a closely related company (e.g. from the same sector), is used for firm-hedging purposes.

We now have four locally risky assets, whose volatility matrix is:

$$\sigma_t = \begin{pmatrix} \sigma_S & \sigma^{B(T-t)} & \sigma^{I(T-t)} & \sigma^V \end{pmatrix}.$$  

A self-financing portfolio strategy is now defined by a vector $w_t$ with four elements, that contains the weights allocated respectively to the stock, the two bonds and the firm-hedging security.

6.2 Dynamic Strategies in Integrated ALM

As long as the terminal asset of the pension fund, $A_T$, is greater than the promised liability payment, $L_T$, no contribution from the sponsor is required, and pensioners...
receive what they were promised to. If, in contrast, $A_T$ is smaller than $L_T$, pensioners may not receive the promised payment in full. Whether they receive it or not depends on the ability of the sponsor to contribute. As long as the assets of the sponsor are in sufficient amount to make up for the deficit of the pension fund plus the face value of corporate debt, a condition which can be written as $V_T > L_T + D - A_T$, the contribution is paid. In the opposite situation, where the level of assets is not sufficient to allow for a full funding of the pension plan, the firm is liquidated, and pensioners only receive a payment equal to $A_T + q(1 - \alpha)VT$, which is less than $L_T$. Hence, what the pension fund wants to avoid in the end is the occurrence of underfunding with a weak sponsor.

6.2.1 A First Example of Floor
Mathematically, such states of the world will be avoided if the following constraint is satisfied:

$$A_T \geq \min (L_T, L_T + D - V_T)$$  \hspace{1cm} (6.1)

We denote with $F^1_T$ the terminal value of the floor written in the right side. The fair value of this floor at date $t$ is given by:

$$F^1_t = \mathbb{E}^Q_t \left[ e^{-\int_t^T r_s \, ds} F^1_T \right].$$

It should be noted that the terminal floor value can be negative with positive probability. If the assets of the sponsor exceed the promised payment to pensioners and debtholders, then $F^1_T$ equals $L_T + D - V_T$, which is a negative quantity. As a consequence, the price $F^1_T$ can be negative. Because weights that relate to a negative wealth are hard to interpret, we will not express the floor-replicating strategy in terms of weights, but instead in terms of dollar amounts invested in the various traded securities. Let $\theta^{F,1}_t$ denote the vector of dollar amounts, with the elements ordered as follows:

$$\theta^{F,1}_t = \begin{pmatrix} \text{stock index} \\ \text{nominal bond} \\ \text{indexed bond} \\ \text{firm-hedging security} \end{pmatrix}.$$

In case the floor $F^1_t$ is positive, one can recover the vector of weights as:

$$w^{F,1}_t = \frac{1}{F^1_t} \theta^{F,1}_t.$$

The following proposition gives an explicit expression for the floor value, and for the floor-replicating portfolio.

**Proposition 6.1** The floor at date $t$ is given by:

$$F^1_t = L_t - V_T \mathcal{N} \left( d^{V,t}_1 \right) + D^{\mathcal{N}} \mathcal{N} \left( d^{V,t}_2 \right),$$

where:

$$d^{V,t}_1 = \frac{1}{\sqrt{\int_t^T \| \sigma^V - \sigma^B(T-s) \|^2 \, ds}} \left[ \ln \frac{V_t}{D^t} + \frac{1}{2} \int_t^T \| \sigma^V - \sigma^B(T-s) \|^2 \, ds \right],$$

$$d^{V,t}_2 = \frac{1}{\sqrt{\int_t^T \| \sigma^V - \sigma^B(T-s) \|^2 \, ds}}.$$

The floor-replicating portfolio reads:

$$\theta^{F,1}_t = L_t u_3 - V_t \beta_{t} \mathcal{N} \left( d^{V,t}_1 \right) u_3 + D^{\mathcal{N}} \mathcal{N} \left( d^{V,t}_2 \right) u_2.$$
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where the vectors $u_2$, $u_3$ and $u_4$ denote respectively a portfolio fully invested in the nominal zero-coupon, a portfolio fully invested in the indexed one, and a portfolio fully indexed in the firm-hedging security:

$$
\begin{align*}
  u_2 &= \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \\
  u_3 &= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \\
  u_4 &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.
\end{align*}
$$

Proof. See appendix A.4.

We recall that $D_t^P$, the present value of the promised payment to bondholders, is defined in (3.2). The floor-replicating portfolio can be simplified if interest rates are constant and liabilities are not indexed on inflation, thus making the nominal and the indexed zero-coupon bonds unnecessary. In this case, we obtain:

$$
\theta_t^{F,1} = -V_t \beta_t \mathcal{N}(d_{1,t}^V) u_4,
$$

where $u_4$ is a portfolio fully invested in the firm-hedging security.

Having computed the floor-replicating portfolio, we can now consider strategies that guarantee that $A_t$ stays above the floor $F_t$ at all dates. A family of such strategies is given by:

$$
 w_t = m \left( 1 - \frac{F_t^1}{A_t} \right) w_t^u + \frac{1}{A_t} \theta_t^{F,1},
$$

for some unconstrained strategy $w_t^u$ and some multiplier $m$. The proof given in appendix A.1 shows that the terminal wealth generated by this strategy satisfies constraint (6.1).

6.2.2 Other Possible Floors

The floor $F^1$ defined in (6.1) has one drawback: it can become negative with positive probability. Indeed, it is negative as soon as $V_T$ exceeds the sum of promised payments to pensioners and bondholders. In particular, even if the constraint (6.1) is satisfied, wealth may still be negative. Because it is undesirable for the pension fund to have negative wealth, one can consider the following modified floor:

$$
F_t^2 = \max \left( F_t^1, 0 \right) = \max \left( \min \left( \frac{L_T}{d^V_T}, L_T \right), 0 \right).
$$

Because the market is complete, we know that there exists a portfolio strategy $w_t^{F,2}$ that replicates $F_t^2$. But no analytical expression is available for $w_t^{F,2}$, or for the price $F_t^2$, because the payoff $F_t^2$ is a non-linear function of the payoff $F_t^1$, which is itself non-linear in the random variables $V_T$ and $\Phi_T$.

In contrast with $F^1$, the floor $F^2$ is always positive. On the other hand, it does not allow for an explicit control of the minimum funding ratio, and potentially allows for a zero funding ratio at the terminal date. A floor that avoids situations of large underfunding is:

$$
F_t^3 = \max \left( \min \left( L_T + D - V_T, L_T \right), k L_T \right).
$$

Indeed, if the constraint $A_T \geq F_t^3$ is satisfied with probability 1, then the terminal funding ratio is greater than $k$ with probability 1. This limits the size of the deficit of the pension plan. This floor can be regarded as an hybrid floor that mixes features of the CPPI floors analysed.
from an isolated ALM perspective and floors $F^i_1$ and $F^i_2$, which are consistent with a more integrated ALM perspective. As for $F^2$, there is no analytical expression for the floor-replicating strategy $w^F$. For each of the floors $F^2$ and $F^3$, we then consider the RCI strategy defined as:

$$w_t = m \left(1 - \frac{F^i_1}{A_t}\right) w^F_t + \frac{F^i_2}{A_t} w^F_t.$$  \hspace{1cm} (6.6)

The proof of appendix A.1 can be adapted to show that the wealth generated by this strategy reads:

$$A_T = F^i_1 + (A^0_t)^m A_0 - F^i_2 \exp \left[ \left(1 - m \right) \int_0^T \left( r_s + \frac{\sigma_s w^F_t}{2} \right) \text{d}s \right].$$  \hspace{1cm} (6.7)

Taking $t = T$, this expression allows for a direct simulation of the payoff of the wealth $A_T$ without requiring explicit computation of the floor-replicating portfolio. Both floors $F^2$ and $F^3$ have the following important property, the proof of which is immediate given the definition of the floors.

**Proposition 6.2** Assume that the terminal constraint $A_T \geq F^i_1$, with $i = 2$ or 3, is satisfied. Then pensioners receive the promised payment in all states of the world.

**Proof.** See appendix A.5.

It should be noted that for the constraint $A_T \geq F^i_1$ to be satisfied almost surely, the pension fund must start from an initial capital $A_0$ such that $A_0 \geq F^i_0$.

### 6.3 Numerical Illustration

In figures 17 and 18, we provide a numerical illustration of the properties of the RCI strategies that aim at controlling sponsor risk by respecting either the constraint $A_T \geq F^i_1$ or $A_T \geq F^i_2$ at the terminal date. We compare them to a CPPI strategy that protects a minimum funding level $k = 80\%$. So as to make a fair comparison between the two strategies and avoid the impact of discrete implementation on the distribution in our results, we have simulated the payoff of the CPPI strategy using (4.4), and the payoff of the risk-controlled strategy with sponsor risk hedging using (6.7). Each payoff was simulated 200,000 times, and the fair values of the claims were estimated by Monte-Carlo. We test the floors $F^2$ and $F^3$ in order to assess the impact of buying an insurance against downside risk in addition to an insurance against contribution risk. Indeed, with the floor $F^2$, which is dominated by $F^3$, the pension fund can end with arbitrarily low funding ratios. In contrast, the floor $F^3$ guarantees that the final funding ratio is at least equal to 80\%.

The histograms on the left in figures 17 and 18 show the distribution of the terminal funding ratio when the pension fund implements the basic ALM CPPI strategy. As noted in the previous sections, the distribution of the funding ratio is truncated on the left side, but since the minimum guaranteed funding ratio is only 80\%, the pension fund is left with a shortfall probability of about 40.95\%. In integrated ALM, a situation of underfunding is not a concern in itself, as long as the sponsor can make up for the deficit. Hence, it is more informative to
look at the likelihood of “bad” states of the world, where the pension fund ends up underfunded and the sponsor cannot fill in the gap: this probability, which is mathematically measured by $P(A_t < L_t, A_t + V_t < L_t + D)$, is equal to 10.61%. Such states of the world are highly undesirable from the perspective of all stakeholders: indeed, pensioners receive less than the promised payment, the sponsor firm is put bankrupt, bondholders only receive a recovery payment, and equity holders receive nothing. In contrast, when sponsor risk is taken into account, the joint probability that the pension fund is insolvent and the sponsor is unable to fill in the gap falls to zero, which precisely was the defining property of such integrated dynamic LDI strategies. Consequently, the fair value of the contribution made by the sponsor increases, from $4.66 with the CPPI strategy to $16.35 with the floor $F_2$ and $5.72 with the floor $F_3$. In fact, in both cases, the actual contribution, $C_T$, is equal to the required contribution since the pension deficit never exceeds the financial resources of the sponsor. Another consequence of sponsor risk hedging is that the probability of sponsor default decreases. This probability is mathematically given by $P(A_t + V_t < L_t + D)$, that is the probability that the total assets of the sponsor and pension fund are less than the sum of promised payments. It is equal to 11.30% with the CPPI strategy, and it decreases to 3.42% and 6.73% respectively with the RCI strategies based on floors $F_2$ and $F_3$.

Unsurprisingly, avoiding the states of the world where the pension fund is underfunded and the sponsor cannot pay the required contribution has a positive impact on the value of pension claims: it increases from $46.56 with the CPPI strategy, to $48.35 with both floors. The
fact that the present value of pension claims is the same for both floors $F^2$ and $F^3$ and that it is higher than for the CPPI strategy is explained by proposition 6.2. Pensioners receive the promised payment in all states of the world, so the fair value of their claims equals the value of a default-free indexed bond. Bondholders also benefit from the RCI strategy: the value of corporate bonds increases from $27.25 to $30.75 with the floor $F^2$ and $30.04 with F$^3$. This increase comes from the decrease in the probability of the “bad” states of the world: the probability increases of having either the pension fund solvent ($A_t \geq L$) or having the aggregate asset of the firm and pension fund exceed the promised payments ($A_t + V_t \geq L_t + D$). In the first case, there is no need for the sponsor to contribute, so its assets are fully available to redeem debt. In the second case, bondholders always receive the promised payment because the sponsor’s deficit is covered by the surplus of the pension fund. We actually found that the likelihood of this second situation increases substantially when sponsor risk is managed: it grows from 88.70% with the CPPI strategy to 96.58% with the floor $F^2$ and 93.27% with $F^3$.

The impact of the RCI strategy that controls sponsor risk on equity value depends on the chosen floor. With the floor $F^2$, equity value increases from $40.85 to $43.23, but with the floor $F^3$, it decreases to $39.66$. The fact that it decreases with the floor $F^3$ is straightforward: the terminal value $F^3_T$ is always greater than or equal to the terminal floor of the CPPI strategy, which is equal to $kL$. As a consequence, the price to pay for the payoff $F^3_T$ is higher than the price for the payoff $kL$.

The difference is the additional cost of the insurance against sponsor risk, a cost that reduces the access to upside performance for equity holders. But when the floor $F^2$ is used, equity value increases. The explanation is similar: with our parameter values, the payoff $F^2_T$ is less expensive than the payoff $kL$, so equity holders have more access to upside with the RCI strategy that controls sponsor risk than with the CPPI.

Finally, it can be noted that the total value of the firm, which can be interpreted as some aggregate measure of welfare for the collection of all stakeholders, increases substantially when the integrated ALM perspective is favoured. It is $114.66 with the CPPI strategy, and grows to $122.34 and $118.05 respectively with the floors $F^2$ and $F^3$. This increase comes from a strong reduction in the present value of bankruptcy costs related to the decrease in the probability of default. Indeed, as previously mentioned, the probability that $A_T + V_T \geq L_T + D$, a situation in which the sponsor does not default because its assets cover the promised payoff to bondholders by themselves or the surplus of the pension fund covers the gap, is higher with the integrated dynamic LDI strategies than with the basic CPPI strategy.

In figure 19, we specifically look at the impact of the multiplier on the values of the claims, when the pension fund implements the RCI strategy (6.6) either with the floor $F^2$ or with the floor $F^3$. The qualitative effect of the multiplier with such RCI strategies is similar to the effect obtained with the basic CPPI: pensioners are indifferent to the choice of the
multiplier because they receive the promised payment and no more in all states of the world, equity holders benefit from a more aggressive policy, and bondholders are hurt by an increase in the multiplier. But the RCI strategy with a control of sponsor risk appears to lead to a higher value for each claim, compared to the basic CPPI. The only exception is with the most expensive floor, that leads to lower equity value. However, the improved RCI strategy leads to a higher total value than a naive strategy that would only invest in the LHP or in the stock index. This is an improvement over the basic CPPI, which has been shown to decrease the total value in many cases (see figure 1 for instance).

6.4 The Incomplete Market Case
In practice, it may prove unfeasible to have access to a security/strategy that can perfectly hedge against the impact of unexpected changes in the underlying firm value. For example, if the dynamic strategy that replicates the process $V$ involves a long position in the equities of the sponsor, then the floor-replicating portfolio involves a short position in this asset. In practice, such short positions might be prohibited, so these strategies may not be directly implementable. More generally, even if shorting the stock of the sponsor company is a feasible strategy for the pension fund, the process $V$ might still be hard to replicate in practice because it would require implementing a complex dynamic trading strategy involving a number of securities (stock of the firm, stock index, inflation-linked bonds). In this context, it is in fact more realistic to assume that the pension fund cannot always in practice build a dynamic strategy that is perfectly correlated with $V$ at all dates, which makes the market incomplete. The best that the pension fund can do is to invest in a security, for example the stock

Figure 18: RCI strategy in ALM with insurances against contribution risk and downside risk – Perfectly hedgeable sponsor risk.
This figure shows the distribution of the terminal funding ratio in two situations: (1) left column: the pension fund implements a CPPI strategy with a minimum funding ratio $k = 80\%$ and a multiplier $m = 3$; (2) right column: it implements a RCI strategy that provides an insurance against contribution risk but no insurance against downside risk (floor $F_3$ in (6.5)). Also reported are the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$), and the following probabilities: the shortfall probability ($P(A_T < L_T)$), the probability that the debt to pensioners and bondholders exceeds the available assets ($P(A_T + V_T < L_T + D_T)$), and the probability of a joint occurrence of the previous two situations ($P(A_T < L_T, A_T + V_T < L_T + D_T)$). Pension liabilities are unconditionally indexed on inflation, the initial funding ratio $R_0$ is $90\%$, and other parameters are set to their base case values (see table 1).
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Figure 19: Impact of the multiplier with RCI strategies that control sponsor risk. Perfectly hedgeable sponsor risk.
This figure shows the values of the claims for different dynamic strategies. The dashed lines represent a situation where the pension fund invests in the LHP only; the dotted lines correspond to an investment in the stock index only; the solid lines with no markers represent the standard CPPI strategy implemented with a minimum funding ratio $k = 80\%$ and a multiplier $m$; the solid lines with triangles represent the RCI strategy with an insurance against contribution risk but no insurance against downside risk (floor $F_2$ in (6.4)); the solid lines with circles represent the RCI strategy that provides insurances against contribution risk and against downside risk (floor $F_3$ in (6.5)). Pension liabilities are unconditionally indexed on inflation, the initial funding ratio $R_0$ is $90\%$, and other parameters are set to their base case values (see table 1).

6.4.1 An imperfect firm-hedging security

Introducing an asset to the economy, particularly one that is not redundant with existing assets and that does not perfectly correlate with $V$, will always involve introducing a new source of risk. For notational consistency, we thus have to redefine the notation introduced in section 2. We assume that the firm-hedging security evolves as:

$$\frac{dG_t}{G_t} = \left[r_t + \sigma^G \lambda_G\right] dt + \sigma^G dZ^G_t, \quad (6.8)$$

issued by a comparable company, that is highly correlated with $V$. We thus replace in this subsection the perfect firm-hedging security by an imperfect one, the value of which we denote with $G$. 

$$R/k = 1.13, \quad R_0 = 0.90$$

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where the Brownian motion \( z^G \) is not redundant with the Brownian motions introduced previously, namely \( z^r, z^\Phi, z^S \) and \( z^V \). The vector form of the model involves a five-dimensional Brownian motion \( \mathbf{z} \), and five volatility vectors \( \sigma^r, \sigma^\Phi, \sigma^S, \sigma^V \) and \( \sigma^G \) such that:

\[
(\sigma^j)\, dz_t = \sigma^j \, d\mathbf{z}_t, \quad \text{for } j = r, \Phi, S, V, G.
\]

The unit volatility matrix \( \mathbf{R} \) of all risks is:

\[
\mathbf{R} = \begin{pmatrix}
\sigma^r & \sigma^\Phi & \sigma^S & \sigma^V & \sigma^G \\
\sigma^r & \sigma^\Phi & \sigma^S & \sigma^V & \sigma^G \\
\sigma^r & \sigma^\Phi & \sigma^S & \sigma^V & \sigma^G \\
\sigma^r & \sigma^\Phi & \sigma^S & \sigma^V & \sigma^G \\
\sigma^r & \sigma^\Phi & \sigma^S & \sigma^V & \sigma^G
\end{pmatrix}
\]

and the market price of risk vector is now given by:

\[
\mathbf{\Lambda} = \mathbf{R} \left( \mathbf{R}^T \mathbf{R} \right)^{-1} \begin{pmatrix}
\lambda^r \\
\lambda^\Phi \\
\lambda^S \\
\lambda^V \\
\lambda^G
\end{pmatrix}
\]

The pricing kernel implied by the market is then still given by (2.5), with the new vector \( \mathbf{\Lambda} \) and the five-dimensional Brownian motion \( \mathbf{z} \). Because \( G \) is not perfectly correlated with \( V \), the correlation \( \rho^{GV} \) is neither equal to \(-1\) or \(+1\). In what follows, we shall assume that \( G \) has been chosen in such a way that it offers a positive and significant correlation with \( V \), and we consider values for \( \rho^{GV} \) that are taken to be either 60% or 80%. The imperfect correlation is not the only reason why \( G \) and \( V \) differ: they can also have different volatilities and different Sharpe ratios. In order to focus on the sole impact of an increase in the correlation, we will, however, assume that their volatilities are equal, and that their Sharpe ratios are equal too. With this assumption, we have the following result, the proof of which is given in appendix A.6:

**Proposition 6.3** The quadratic replication error in the approximation of the non-replicable payoff \( V_T \) by the replicable payoff \( G_T \) is:

\[
\mathbb{E} \left[ \left( \ln \frac{G_T}{V_T} \right)^2 \right] = \left( \ln \frac{G_0}{V_0} \right)^2 
+ \left( \sigma^V \right)^2 \left[ 1 - \left( \rho^{GV} \right)^2 \right] T.
\]

This proposition shows that in order to minimise the replication error, one should be investing the capital \( V_0 \) in the firm-hedging security, and, of course, take the correlation to be as high as possible. In what follows, we will assume that \( G_0 = V_0 \), so the residual replication error is entirely due to the imperfect correlation between \( G \) and \( V \), and we will let the correlation parameter \( \rho^{GV} \) vary.

### 6.4.2 Different floors

The objective is in fact not to replicate \( V_T \) itself, but instead the terminal values of the floors \( F^1, F^2 \) and \( F^3 \) introduced in the previous subsection, which are non-linear functions of \( V_T \). However, since \( V_T \) is not replicable, the payoffs \( F^1_T, F^2_T \) and \( F^3_T \) cannot be replicated perfectly. On the other hand, one can still replicate the following payoffs, which are obtained by replacing \( V_T \) by \( G_T \) in \( F^1, F^2 \) and \( F^3 \):

\[
\tilde{F}^1_T = \min(L_T, L_T + D - G_T), \\
\tilde{F}^2_T = \max(0, \min[L_T, L_T + D - G_T]), \\
\tilde{F}^3_T = \max(kL_T, \min[L_T, L_T + D - G_T]).
\]

(6.9)

Two facts are worth noting. First, the payoff \( \tilde{F}^1_T \) does not super-replicate the payoff \( F^1_T \). Hence, even if the terminal wealth satisfies \( A_T \geq \tilde{F}^1_T \) with probability 1, there may still be states of the world where \( A_T < F^1_T \). Second, we are not arguing that
6. Insuring Against Sponsor Risk

\( \tilde{F}_T^i \) is the “best approximation” of \( F_T^i \) in any sense: \( G_T \) is a good approximation of \( V_T \) in the sense of the quadratic error defined in proposition 6.3 if the correlation \( \rho^{GV} \) is high, but it does not follow that \( \tilde{F}_T^i \) is a good approximation of \( F_T^i \) in the sense of quadratic error or in any other sense. The general problem of pricing the best approximation of the payoffs \( F_T^1, F_T^2, \) and \( F_T^3 \) is a problem of option replication in incomplete market, which is beyond the scope of this paper. Nevertheless, the payoffs \( \tilde{F}_T^1, \tilde{F}_T^2, \) and \( \tilde{F}_T^3 \) are replicable and are likely to be often close to the payoffs \( F_T^1, F_T^2, \) and \( F_T^3, \) as long as the security \( G \) is reasonably close to \( V. \)

Denoting the strategy that replicates the payoff \( \tilde{F}_T^i \) with \( w_t^{F,i}, \) we define the dynamic integrated LDI strategies as:

\[
\omega_t = m \left( 1 - \frac{\tilde{F}_t^i}{A_t} \right) \omega_t^0 + \frac{\tilde{F}_t^i}{A_t} \omega_t^{F,i}, \tag{6.10}
\]

where \( \omega_t^0 \) is some unconstrained strategy and \( m \) is some multiplier. The wealth generated by this strategy is given by the usual expression:

\[
A_t = \tilde{F}_t^i + \left( A_t^m \right)^m \frac{A_0 - \tilde{F}_0^i}{A_0^m} \exp \left[ (1 - m) \int_0^t \left( r_s + m \| \sigma_s \| w_t^0 \|^2 \right) ds \right], \tag{6.11}
\]

for \( i = 1, 2, 3. \)

6.4.3 Numerical results

In figures 20 and 21, we perform a comparison between strategy (6.10) and a standard CPPI strategy, assuming that sponsor risk is not fully hedgeable. We consider two values of the correlation \( \rho^{GV}, \) namely 60% and 80%. The second value describes a situation where the firm-hedging security is highly correlated with the sponsor’s unlevered value.

The RCI strategy guarantees that the terminal wealth satisfies \( A_T \geq \tilde{F}_T, \) but it does not ensure that \( A_T \geq F_T. \) Hence the probability of “bad” states of the world, where the pension fund is underfunded and the sponsor cannot make up for the gap, is not zero. But the numbers in the figures show that the probability of the joint occurrence of a weak sponsor and an underfunded pension plan is significantly smaller with the integrated RCI strategy than with the more basic ALM CPPI strategy. For instance, when the CPPI strategy is implemented, the probability is as high as 10.75%, but if the pension fund follows the RCI strategy with floor \( F^2, \) the probability falls to 5.90% when \( \rho^{GV} \) is 60%. Of course, when the firm-hedging security better hedges sponsor risk, the decrease is even more pronounced, and the probability falls to 3.28%. Similar observations can be made for the floor \( F^3. \) We also note that the probability of default tends to decrease as well: it is 11.42% with the CPPI strategy, but it is less with the RCI strategy, provided the correlation \( \rho^{GV} \) is sufficiently high.

Looking at the values of the claims, we observe that pensioners benefit in general from the management of sponsor risk, except if the correlation \( \rho^{GV} \) is only 60%, and the pension fund implements the RCI strategy with the floor \( F^2. \) Indeed, this floor does not provide any explicit protection against funding ratio downside risk, so the pension fund may be left with very low funding ratios, and the sponsor may then be unable to make up for such large gaps. Such situations occur because sponsor risk is no longer perfectly hedged, so the total asset \( A_T + V_T \) can be less than the total liabilities \( L_T + D \) even with the RCI
strategy (6.10). On the other hand, if the pension fund chooses the floor $F^3$, which provides an insurance against downside risk, then, the fair value of pension claims is higher than with the CPPI strategy.

For both floors and both correlation levels, the value of corporate bonds increases if the pension fund takes the RCI strategy. The largest increase, however, takes place when the strategy is implemented with the floor $F^2$, which is the less expensive of the two floors. Indeed, a less expensive floor allows for extended access to the upside of the unconstrained strategy, hence it opens the possibility for larger surpluses. Bondholders benefit from these surpluses, since surpluses can help the

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Figure 20: RCI strategy in ALM with insurance against contribution risk and no insurance against downside risk – Partially unhedgeable sponsor risk. This figure shows the distribution of the terminal funding ratio in two situations: (1) left column: the pension fund implements a CPPI strategy with a minimum funding ratio $k = 80\%$ and a multiplier $m = 3$; (2) right column: it implements a RCI strategy that provides an insurance against contribution risk but no insurance against downside risk (floor $F^2$ in (6.9)). Also reported are the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$), and the following probabilities: the shortfall probability ($P(A_t < L_t)$), the probability that the debt to pensioners and bondholders exceeds the available assets ($P(A_t + V_t < L_t + D)$), and the probability of a joint occurrence of the previous two situations ($P(A_t < L_t, A_t + V_t < L_t + D)$). Pension liabilities are unconditionally indexed on inflation, the initial funding ratio $R_0$ is 90\%, and other parameters are set to their base case values (see table 1).

(a) Correlation between firm-hedging security and firm value $\rho^{GV} = 60\%$.

(b) Correlation between firm-hedging security and firm value $\rho^{GV} = 80\%$. 
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Sponsor to pay back the debt. In a similar way, equity value is higher when the floor $F^2$ is preferred to $F^3$ by the pension fund.

Finally, there appears to be a trade-off. Pensioners are better off with a floor that avoids large shortfalls, such as the floor $F^3$, while bondholders and equity holders, who have access to surpluses, prefer a less expensive floor, such as $F^2$. Overall, the total value increases more with the floor $F^2$, which suggests that this floor should be preferred from a collective perspective. This conclusion is the same as in the complete market case (see the previous subsection), where it was shown that the total value increased more when the pension fund did not insure against downside risk.

Figure 21: RCI strategy in ALM with insurances against contribution risk and downside risk – Partially unhedgeable sponsor risk. This figure shows the distribution of the terminal funding ratio in two situations: (1) left column: the pension fund implements a CPPI strategy with a minimum funding ratio $k = 80\%$ and a multiplier $m = 3$; (2) right column: it implements a RCI strategy that provides insurances against contribution risk and downside risk (floor $F^3$ in (6.9)). Also reported are the values of the claims held by pensioners ($P_0$), equity holders ($E_0$) and debtholders ($D_0$), as well as the total value ($v_0$), and the following probabilities: the shortfall probability ($P(A_T < L_T)$), the probability that the debt to pensioners and bondholders exceeds the available assets ($P(A_T + V_T < L_T + D)$), and the probability of a joint occurrence of the previous two situations ($P(A_T < L_T, A_T + V_T < L_T + D)$). Pension liabilities are unconditionally indexed on inflation, the initial funding ratio $R_0$ is 90\%, and other parameters are set to their base case values (see table 1).

(a) Correlation between firm-hedging security and firm value $\rho_{GV} = 60\%$.

(b) Correlation between firm-hedging security and firm value $\rho_{GV} = 80\%$. 

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In figure 22, we study how the values of the claims vary with the multiplier, choosing the pessimistic value of 60% for the instantaneous correlation between the unlevered value of the sponsor and the firm-hedging security. The impacts are qualitatively the same as in the case where sponsor risk was perfectly hedgeable, but it appears that the strategy that leads to the highest total value is the RCI strategy implemented with the floor $\tilde{F}^3$: it is the one that not only attempts to control sponsor risk through an investment in the firm-hedging security, but also controls downside risk by imposing a minimum terminal funding constraint. These findings may provide support for the incorporation of formal minimum funding requirements in the dynamic strategy of the pension fund.

Figure 22: Impact of the multiplier with RCI strategies that control sponsor risk – Partially unhedgeable sponsor risk.
This figure shows the values of the claims for different dynamic strategies. The dashed lines represent a situation where the pension fund invests in the LHP only; the dotted lines correspond to an investment in the stock index only; the solid lines with no markers represent the standard CPPI strategy implemented with a minimum funding ratio $k = 80\%$ and a multiplier $m$; the solid lines with triangles represent the RCI strategy with an insurance against contribution risk but no insurance against downside risk (floor $\tilde{F}^2$ in (6.9)); the solid lines with circles represent the RCI strategy that provides insurances against contribution risk and against downside risk (floor $\tilde{F}^3$ in (6.9)). The correlation $\rho_{GV}$ between the unlevered asset of the sponsor and the firm-hedging security is set to 60%. Pension liabilities are unconditionally indexed on inflation, the initial funding ratio $R_0$ is 90%, and other parameters are set to their base case values (see table 1.)
Conclusion
Conclusion

The optimal management of pension assets is made complex by the presence of fundamental conflicts of interest between various stakeholders, shareholders and pensioners. If it is assumed that they do not have access to any pension fund surplus, then risk taking is detrimental from their perspective, because it involves increasing the likelihood of partial recovery of pension claims, whereas the upside potential of the performance-seeking assets allows shareholders to reduce the burden of contributions needed to meet expected pension payments. Within the context of a formal capital structure model, which allows for the rational valuation of liability streams as collateralised, non-tradable, defaultable claims issued by the sponsor company, Martellini and Milhau (2010a) have argued that an effective way to align the incentives of shareholders and pensioners without any complex adjustment to the pension plan structure consists of enlarging the set of admissible investment strategies so as to include dynamic risk-controlled strategies.

While providing a first step towards a better understanding of the benefits of risk-controlled dynamic asset allocation strategies from an integrated ALM perspective, these preliminary results, however, are based on relatively basic forms of dynamic allocation strategies. This paper provides a formal analysis of the benefits that would arise from a variety of more complex dynamic liability-driven investing strategies designed to maximise stakeholders’ welfare within an integrated asset-liability management context. We find that implementing risk-controlled strategies aimed at insuring a minimum funding ratio level above a minimum value allows shareholders to get some (limited) access to the upside performance of risky assets, while ensuring that pensioners will not be overly hurt by the induced increase in risk. We also find that imposing a cap on the terminal funding ratio allows for a decrease in the cost of downside risk protection, which has a positive impact on pensioners and bondholders without an overly significant decrease in equity value for reasonable parameter values. Finally, we test strategies that aim to control sponsor risk by providing insurance against states of the world characterised by the joint occurrence of an underfunded pension plan and a weak sponsor company. Under most circumstances, these strategies are found to increase pensioners’ and bondholders’ welfare compared to basic CPPI strategies adapted to the ALM context. Shareholders’ welfare is not necessarily increased by these strategies, because some access to the upside performance has to be given up in exchange for the insurance against sponsor risk, but the decrease in equity value is always limited in magnitude. It is always compensated by the increases in the values of pension claims and corporate bonds, in the sense that the total value of the firm and pension fund is higher if sponsor risk is managed by the pension fund through a dynamic strategy than if it is simply disregarded. Overall, our findings suggest that suitably-designed dynamic portfolio strategies can prove to be a very effective answer to some key challenges currently faced by corporate pension plans.
Appendices

A. Proofs

A.1 Wealth generated by CPPI strategies
We consider a self-financing portfolio evolving subject to the budget constraint (3.1), with the portfolio strategy (4.3). We first note that since the LHP perfectly replicates the liabilities, the value of liabilities evolves as:

$$\frac{dL_t}{L_t} = \left[ r_t + \left( w_t^{\text{LHP}} \right)' \sigma_t' \right] dt + \left( w_t^{\text{LHP}} \right)' \sigma_t' dz_t.$$  

Hence the dynamics of the risk budget reads:

$$d[A_t - kL_t] = dA_t - k dL_t$$

$$= A_t - kL_t r_t dt + \left[ A_t, w_t - kL_t, w_t^{\text{LHP}} \right]' \sigma_t' [\lambda dt + dz_t]$$

$$= A_t - kL_t \left[ r_t dt + m (w_t)' \sigma_t' [\lambda dt + dz_t] \right].$$

Moreover, an application of Ito’s lemma shows that:

$$d\frac{A_t - k L_t}{(A_t^0)^m} = \frac{A_t - k L_t}{(A_t^0)^m} \left[ \frac{d[A_t - k L_t]}{A_t - k L_t} - \frac{m dA_t^0}{A_t^0} \right]$$

$$+ \frac{m(m + 1)}{2} \frac{d(A_t^0)}{(A_t^0)^2} - \frac{m}{(A_t^0)^2} \left[ \frac{d(A_t - k L_t)}{A_t - k L_t} - \frac{m dL_t}{L_t} \right].$$

Hence:

$$d\frac{A_t - k L_t}{(A_t^0)^m} = \frac{A_t - k L_t}{(A_t^0)^m} \left[ (1 - m) r_t + \frac{m(1 - m)}{2} \| \sigma_t w_t \|^2 \right] dt.$$ 

Integrating from date 0 to t, we obtain:

$$\frac{A_t - k L_t}{(A_t^0)^m} = \frac{A_0 - k L_0}{(A_0^0)^m} \exp \left[ (1 - m) \int_0^t \left( r_s + m \frac{\| \sigma_t w_t \|^2}{2} \right) ds \right],$$

which is (4.4).

A.2 Optimality of CPPI strategies
The utility maximization program reads:

$$\max_w \frac{1}{1 - \gamma} \mathbb{E} \left[ (R_T - k)^{1 - \gamma} \right].$$  

The idea of the “martingale approach” (see Cox and Huang (1989)) is to map this dynamic program into a static program where the control variable is the terminal wealth. As explained in subsection 2.1, the pension fund faces an incomplete market situation because it cannot perfectly hedge the source of risk $z^V$. Hence the program (A.1) must be solved using the extension of the martingale approach developed by He and Pearson (1991) (henceforth, HP91).

We consider the market that includes only the bonds and the stock. It is incomplete because $z^V$ is not fully hedgeable, so there exist infinitely many price of risk vectors. As shown by HP91, they are of the form $(\lambda + \nu_t)_t$, where $\nu_t$ is such that $\rho' \nu_t = 0$ at all dates $t$. HP91 show that the original dynamic program (A.1) is equivalent to a static program

$$\max_{A_T} \frac{1}{1 - \gamma} \mathbb{E} \left[ (R_T - k)^{1 - \gamma} \right],$$

subject to $\mathbb{E} [M_T^* A_T] = A_0.$  

where $M^*$ is the “minimax” pricing kernel, given by:

$$M_T^* = \exp \left[ - \int_0^T r_s ds - \int_0^T \frac{\| \lambda \|^2 + \| \nu^* \|^2}{2} ds \right.$$

$$- \int_0^T (\lambda + \nu^*)_t dz_s \].$$

The first-order optimality conditions in (A.2) lead to the optimal terminal wealth:

$$A_T^* = k L_T + \eta^{-1} (M_T^*)^{-1} L_T^{-1} \gamma,$$

where $\eta$ is the Lagrange multiplier associated with the budget constraint.
Hence the optimal wealth process:

\[ A_t^* = \frac{1}{M_t^*} \mathbb{E}_t \left[ kL_T + \eta^{-\frac{1}{\gamma}} (M_T^*)^{-\frac{1}{\gamma}} L_T^{-\frac{1}{\gamma}} \right] \]

\[ = k L_T + \eta^{-\frac{1}{\gamma}} (M_t^*)^{-\frac{1}{\gamma}} L_t^{-\frac{1}{\gamma}} \mathbb{E}_t \left[ \frac{M_T^* L_T}{M_t^* L_t} \right]^{1-\frac{1}{\gamma}}. \]

The process \( L^* \) evolves as:

\[ \frac{d}{M_t^* L_t} = \left[ \sigma_t w_t^{\text{HP}} - \lambda - \nu_t^* \right]^{\gamma} dz_t. \]

Conjecturing that \( \nu_t^* \) is deterministic, we obtain that \( G_t \) is a function of time only. Hence the volatility vector of \( A^* \):

\[ \sigma_t^A = \frac{k L_T}{A_t^*} w_t^{\text{HP}} \]

\[ + \frac{A_t^* - k L_T}{A_t^*} \left[ \frac{1}{\gamma} (\lambda + \nu_t^*) \right] \]

\[ + \left( 1 - \frac{1}{\gamma} \right) \sigma_t w_t^{\text{HP}}. \]

By definition, the volatility vector of wealth is spanned by the columns of \( \sigma_0 \), hence also spanned by the columns of the unit volatility matrix \( \rho \). Hence:

\[ I_3 - \rho (\rho' \rho)^{-1} \rho' \] \( \sigma_t^A = 0, \)

where \( I_3 \) is the identity matrix of size 3. This leads to \( \nu_t^* = 0 \), so \( M^* \) is in fact equal to the minimal pricing kernel \( M \).

Conversely, let us consider the payoff

\[ X_T = k L_T + \eta^{-\frac{1}{\gamma}} (M_T) L_T^{-\frac{1}{\gamma}} \]

where \( \eta \) is adjusted so that \( \mathbb{E}[M_T A_T] \) equals \( A_0 \). The previous discussion shows that \( X_T \) is replicable. Moreover, for any terminal wealth \( A_T \), we have \( \mathbb{E}[M_T A_T] = A_0 \), hence:

\[ \mathbb{E} \left[ \frac{1}{1-\gamma} \left( \frac{A_T}{L_T} - k \right)^{1-\gamma} \right] \leq \mathbb{E} \left[ \frac{1}{1-\gamma} \left( X_T - k \right)^{1-\gamma} \right]. \]

Hence \( X_T \) is indeed the optimal terminal wealth in the original program (A.1), which justifies the conjecture that \( \nu^* \) is a deterministic process.

The solution to the original program (A.1) is the dynamic strategy \( w^* \) given by:

\[ w_t^* = (\sigma_t^A)^{-1} \sigma_t^A \]

\[ = Q_t^{-1} (\rho' \rho)^{-1} \rho' \sigma_t^A. \]

### A.3 Limit behaviour of OBPI strategy with floor and cap

In this appendix we formally show that the strategy that explicitly incorporates the minimum and the maximum funding constraints is fully invested in the LHP when the funding ratio approaches the floor or the cap. We begin with the formal expression of the strategy that maximises expected CRRA utility from \( R_T \) subject to the constraint that \( k \leq R_T \leq k' \). As shown by MM2010b, it is given by:

\[ w_t = \frac{1}{\gamma} \left[ 1 - \frac{k - \mathcal{N}(-d_2, \xi)}{A_t} \right] w_t^{\text{PSP}} \]

\[ + \left( 1 - \frac{1}{\gamma} \right) \left[ 1 - \frac{k' - \mathcal{N}(d_1, \xi)}{A_t} \right] w_t^{\text{HP}}, \]

\( w'' \) is the wealth that is optimal in the absence of any funding constraint, \( A'' \) is the wealth generated by this strategy starting from an initial capital \( A_0 \), and the constant \( \xi' \) is adjusted so as to make the budget constraint hold:

\[ k L_0 \mathcal{N}(-d_2, \xi) + k' L_0 \mathcal{N}(d_1, \xi) \]

\[ + \frac{\xi' A_0}{A_0} \left[ \mathcal{N}(d_1, \xi) - \mathcal{N}(d_1, \xi) \right] = A_0. \]

The terminal wealth generated by (A.3) involves the payoff of a call spread option:

\[ A_T = k L_T + \left[ \xi' A_T'' - k L_T \right]^+ \]

\[ - \left[ \xi' A_T'' - k L_T \right]^+. \]
More generally, one can consider any unconstrained strategy \( w^u \), not necessarily the one that maximises expected CRRA utility, and the wealth generated by this strategy when the initial capital is \( A_0 \). Replacing \( w^* \) and \( A^* \) by \( w^u \) and \( A^u \) respectively in the previous expressions, we obtain another dynamic strategy \( w \) that generates a terminal wealth \( A_T \) such that \( k \leq \frac{A_T}{L_T} \leq k' \). This holds in particular if we take the unconstrained strategy to be fully invested in the stock index. We now examine the limits of \( w_t \) defined by (A.3) when \( R_t \) approaches either \( k \) or \( k' \).

First, we write the wealth generated by (A.3) at date \( t \):

\[
A_t = kL_t + \mathbb{E}_t \left[ M_T \left[ \xi^T A_T - kL_T \right]^+ \right] - \mathbb{E}_t \left[ \left( \xi^T A_T - k'L_T \right)^+ \right].
\]

The difference between the two conditional expectations is the price of the call spread, hence it is strictly increasing in the spot price \( A_T \). Hence \( A_t \) is strictly increasing in the ratio \( R_t^u = A_T^u / L_t \). The constrained wealth can be re-expressed as:

\[
A_t = kL_t \mathbb{N} \left( -d_{2,t} \right) + k'L_t \mathbb{N} \left( d'_{2,t} \right) + \xi^T A_t \left[ \mathbb{N} \left( d_{1,t} \right) - \mathbb{N} \left( d'_{1,t} \right) \right].
\]

Hence, the limits of the ratio \( R_t = A_t / L_t \) as \( R_t^u \) approaches zero or infinity are \( kL_t \) and \( k'L_t \), respectively. Since \( A_t \) is strictly increasing in \( R_t^u \), this implies that:

\[
\lim_{R_t \uparrow k} R_t^u = 0, \quad \lim_{R_t \downarrow k'} R_t^u = \infty.
\]

Substituting these limits into the expressions for \( d_{2,t} \) and \( d'_{2,t} \), we obtain that:

\[
\lim_{R_t \uparrow k} d_{2,t} = -\infty, \quad \lim_{R_t \downarrow k'} d'_{2,t} = \infty.
\]

Hence:

\[
\lim_{R_t \downarrow k} w_t = w_t^{\text{HP}}, \quad \lim_{R_t \downarrow k'} w_t = w_t^{\text{HP}},
\]

which concludes the proof.

### A.4 Floor-Replicating Portfolio in Integrated ALM

In this appendix, we compute the price of the payoff \( F_t \), as well as the replication strategy. Because the terminal floor value can be rewritten as \( F_T = L_T - (V_T - D)^+ \), we have:

\[
F_t = L_t - \mathbb{E}_t \left[ e^{-rT} \mathbb{N} \left( d_{1,t} \right) \right].
\]

The right side involves the price of a call written on \( V \) with a strike price \( D \). The price of this call can be computed by choosing the nominal zero-coupon of maturity \( T \) as the numeraire (see Geman et al. (1995)). This technique allows to apply the Black-Scholes formula, and it leads to (6.2).

It should be noted that because \( F_t \) is less than \( L_t \), we have, by absence of arbitrage opportunities:

\[
F_t \leq L_t \text{ almost surely, for all } t.
\]

In particular, the initial condition \( A_0 \geq F_0 \), that must be satisfied for (6.1) to be satisfied too, is less severe than the condition \( A_0 \geq L_0 \). Hence the terminal constraint (6.1) is especially relevant for pension funds that are initially underfunded. If the initial funding ratio is greater than 100%, then it makes less sense to insure against sponsor risk. Indeed, the pension fund can adopt a CPPI strategy with a minimum funding level at least equal to 100%, as explained in section 4: such a strategy guarantees that pensioners will receive the promised payment at date \( T \). Hence we will only
consider in this section pension funds that are initially underfunded.

The floor-replicating portfolio can be derived from the pricing formula (6.2). Applying Ito’s lemma, we obtain:

\[
\begin{align*}
\frac{dF_t}{F_t} &= (\cdots)dt + L_t\sigma'(T-t)^\prime dz_t \\
&\quad - V_t\mathcal{N}\left(d_{V,t}\right)\sigma'\left(d_{V,t}\right)^\prime dz_t \\
&\quad + D_t^\delta\mathcal{N}\left(d_{V,t}\right)\sigma^\delta(T-t)^\prime dz_t.
\end{align*}
\]

Matching the diffusion terms in both sides, we obtain:

\[
\begin{align*}
\sigma\theta_t^F &= L_t\sigma'(T-t) - V_t\mathcal{N}\left(d_{V,t}\right)\sigma' \\
&\quad + D_t^\delta\mathcal{N}\left(d_{V,t}\right)\sigma^\delta(T-t) \\
&= L_t\sigma'(T-t) - \beta_t V_t\mathcal{N}\left(d_{V,t}\right)\sigma_t^G \\
&\quad + D_t^\delta\mathcal{N}\left(d_{V,t}\right)\sigma^\delta(T-t).
\end{align*}
\]

Multiplying both sides of the latter equality by \((\sigma_t^\prime)^{-1}\theta_t^F\) on the left, we obtain the expression for \(\theta_t^F\).

A.5 Proof of proposition 6.2
We note that if \(A_T \geq F^i_t\) with \(i = 2\) or \(3\), then:

\[A_T \geq F^i_T\]

since both \(F^2_T\) and \(F^3_T\) are greater than \(F^1_T\). Hence \(A_T\) is either greater than \(L_T\), in which case pensioners receive the promised pensions without needing a contribution from the sponsor, or greater than \(L_T + D - V_T\), in which case the sponsor can make up for the deficit of the pension plan. In both cases, pensioners finally receive \(L_T\).

A.6 Proof of proposition 6.3
Applying Ito’s lemma to the dynamics of \(V\) and \(G\) (see (2.4) and (6.8)), we obtain:

\[
\begin{align*}
d\ln G_t - d\ln V_t &= \left[r_t + \sigma^G\lambda^G\right] dt \\
&\quad + \left(\sigma^G\right)^\prime dz_t - \left[r_t + \sigma^V\lambda^V\right] dt - \left(\sigma^V\right)^\prime dz_t.
\end{align*}
\]

Because \(\sigma^G\) and \(\sigma^V\), we thus have that:

\[
d\ln G_t - d\ln V_t = \left(\sigma^G - \sigma^V\right)^\prime dz_t.
\]

Integrating from 0 to \(T\), we get:

\[
\ln \frac{G_T}{V_T} = \ln \frac{G_0}{V_0} + \int_0^T \left[\sigma^G - \sigma^V\right]^\prime dz_t,
\]

hence

\[
\begin{align*}
\mathbb{E}\left[\left(\ln \frac{G_T}{V_T}\right)^2\right] &= \left(\ln \frac{G_0}{V_0}\right)^2 \\
&\quad + 2\left(\ln \frac{G_0}{V_0}\right)\mathbb{E}\left[\int_0^T \left[\sigma^G - \sigma^V\right]^\prime dz_t\right] \\
&\quad + \mathbb{E}\left[\left(\int_0^T \left[\sigma^G - \sigma^V\right]^\prime dz_t\right)^2\right].
\end{align*}
\]

The stochastic integral has zero mean, so the second term in the right side cancels out. Using Ito isometry, we obtain the result.
Appendices
References
References


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¹ Source: BNPP IP, per 31 December 2011
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About EDHEC-Risk Institute

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EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context.

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- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

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<td>Nbr of participants at EDHEC-Risk Institute Executive Education seminars</td>
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