Inquire Europe Autumn 2009 Seminar

Madrid, October 13th, 2009 – 09:45-10:45

Dynamic Allocation Decisions in the Presence of Funding Ratio Constraints

Lionel Martellini

Professor of Finance, EDHEC Business School

Scientific Director, EDHEC Risk and Asset Management Research Center

lionel.martellini@edhec.edu

www.edhec-risk.com

Joint work with Vincent Milhau

This research was carried out as part of the BNP Paribas Investment Partners "Asset-Liability Management and Institutional Investment Management" research chair at EDHEC-Risk Institute

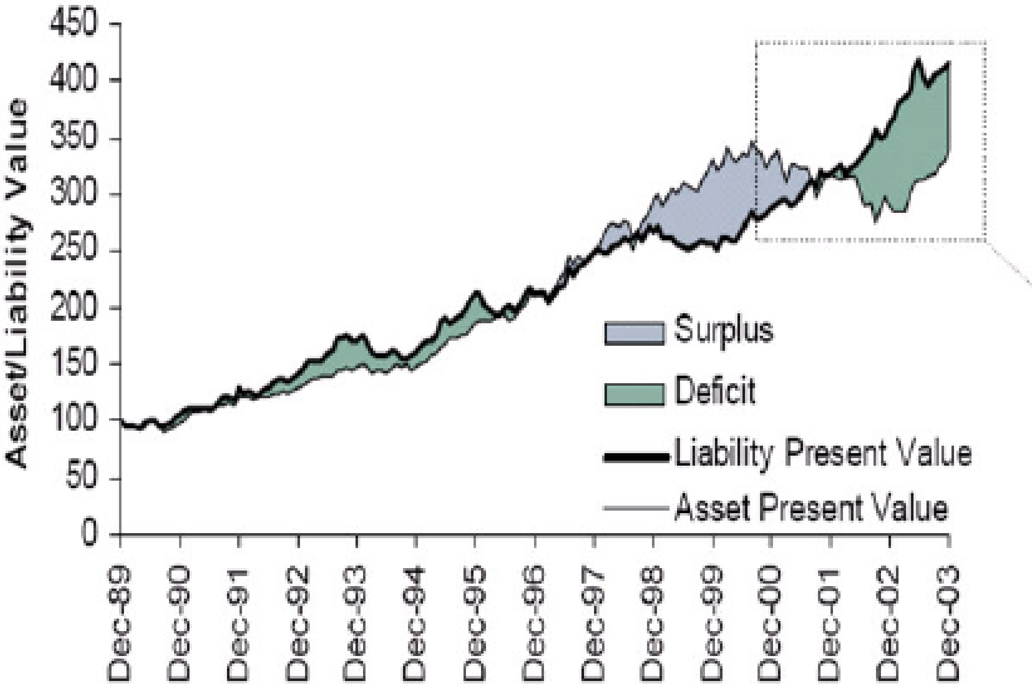
Outline

- Introduction
- From Static to Dynamic LDI Strategies
- Pro-Cyclicality of Risk-Controlled Strategies
- Conclusions and Suggestions for Further Research

- Introduction
- From Static to Dynamic LDI Strategies
- Pro-Cyclicality of Risk-Controlled Strategies
- Conclusions and Suggestions for Further Research

Pension Fund Crisis 2000-2003

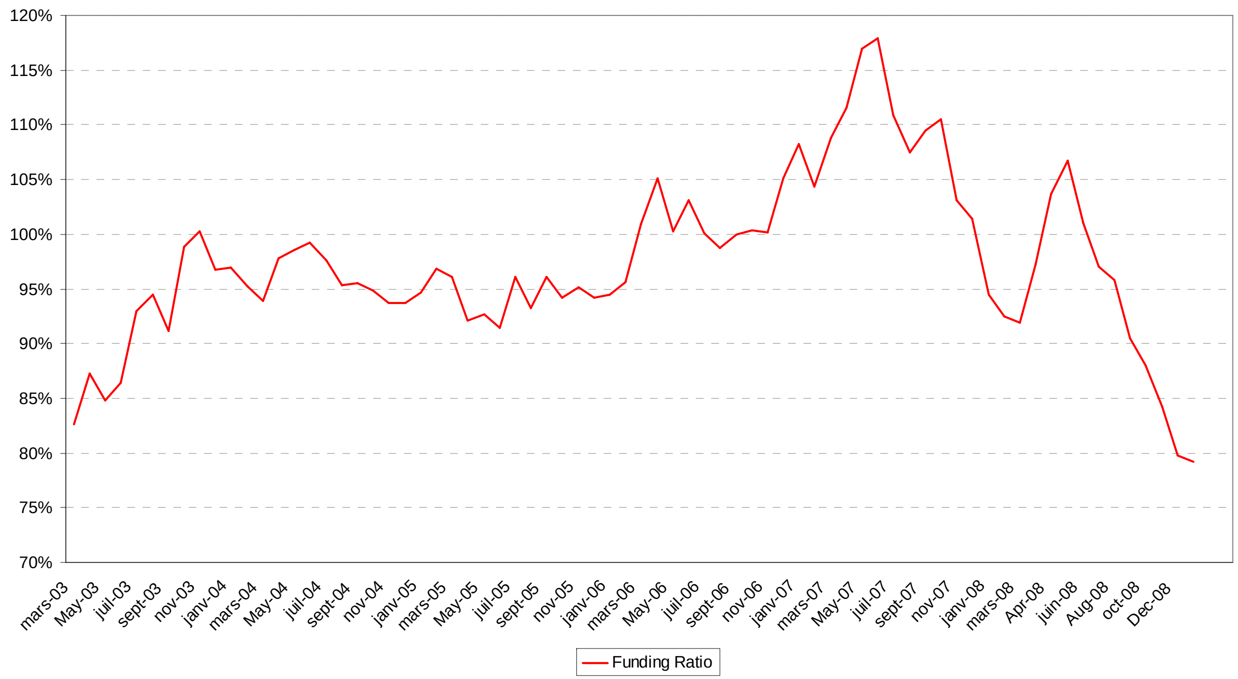
- S&P 500 DB Pension Plans
  - Dec 1999 → May 2003
  - Net surplus of $ 239 billion → Net deficit of $ 252 billion



Pension Fund Crisis – Again?

- In the UK, the aggregate funding ratio at 80%, similar to the US where the aggregate FR is 79% in Dec 08 for top 100 pension funds, versus 109% in Dec 07.

UK Pension Fund aggregate funding ratio (sf 179)



Sources - UK: Pension Protection Fund. 2009. PPF 7800 index, End-March. US: Watson Wyatt, 2009. "U.S. Pension Plan Funding Plunged by More Than $300 Billion in 2008". http://www.watsonwyatt.com/us/pubs/insider/showarticle.asp?ArticleID=20764

Managing Pension Risks

- Defined-benefit pension funds, and the whole pension system more generally, are currently facing a challenge and a dilemma.
- The desire to alleviate the burden of contributions leads them to invest significantly in equity markets and other classes that are poorly correlated with liabilities but offer better long-term performance potential.
- However, stricter regulations and accounting standards give them significant incentives to invest mostly in assets that are highly correlated with liabilities.
- Simply switching into bonds at the expense of equity allocation induces a particularly strong opportunity cost for long-term investors.

State-Dependent ALM Strategies

- Existing ALM techniques that mostly rely on static asset allocation decisions (B&H or FM) can not properly address the complexity of the dilemma.

- In fact, Merton (1969, 1971) has formally shown that static fixed-mix strategies were optimal for a long-term investors if and only if:
  - Constant opportunity set: rules out for example the presence of mean-reversion in equity returns (hedging).
  - No risk constraints: investors care only about wealth at horizon, without minimum values (insurance).

- We will argue that in general only dynamic strategies can help long-term investors such as pension funds meet their goals in the presence of time-varying opportunity sets and risk constraints (our main focus).

- Introduction
- From Static to Dynamic LDI Strategies
- Pro-Cyclicality of Risk-Controlled Strategies
- Conclusions and Suggestions for Further Research

From AM to ALM

- Asset-liability management:

$$\underset{w_s, t \le s \le T}{Max} E_t\left[u\left(F_T \equiv A_T / L_T\right)\right] \quad \text{or} \quad \underset{w_s, t \le s \le T}{Max} E_t\left[u\left(S_T \equiv A_T - L_T\right)\right]$$

  – In ALM, what matters is not asset value per se, but asset value relative to liability value, measured either in terms of funding ratio or surplus.

  – We first consider a simple (benchmark) setting with no short-term funding constraints and a constant equity risk premium set; on the other hand, we consider a Vasicek process for the real rate and (realized) inflation.

  – Asset mix: stocks + nominal & real bonds (complete markets).

Static LDI Solutions

- Optimal unconstrained strategy (with constant risk premium λ):

$$w_t^{*u} = \frac{1}{\gamma}\underbrace{\sigma_t^{-1}\lambda}_{PSP} + \left(1-\frac{1}{\gamma}\right)\underbrace{\left[\underbrace{\sigma_t^{-1}\sigma_\Phi}_{IHP} + \underbrace{\sigma_t^{-1}\sigma_B}_{IRHP}\right]}_{LHP}$$

- We thus obtain a “fund separation theorem”, a.k.a. LDI:
  - The first portfolio is the standard MSR performance-seeking portfolio, already present in asset-only decisions.
  - The second portfolio is a liability-hedging portfolio: it can be shown to have the highest correlation with the liabilities; alternatively, it is a portfolio, invested in cash or derivatives instruments, that minimizes the local volatility of the funding ratio,

Accounting for FR Constraints

- The previous allocation strategy was not explicitly taking the presence of MFR constraints into account.

- We now introduce explicit funding ratio constraints:

$$\underset{w_t,0\le t\le T}{Max}\; E\left[u\left(A_T \big/ L_T\right)\right] \text{ such that } A_t \big/ L_t \ge k \text{ for all } t \in \left[0,\mathrm{T}\right]$$

- The resulting allocation strategy will have to evolve as a function of the current funding ratio, the key state variable, so as to ensure for the respect of the risk constraint.

Dynamic LDI Strategies

- Optimal portfolio strategy (constant λ so far):

$$w_t^{*k} = \frac{1}{\gamma}\underbrace{\left(1 - kp_{t,T}\frac{L_t}{A_t^{*c}}\right)}_{\text{risk budget}}\sigma_t^{-1}\lambda + \left(1 - \frac{1}{\gamma}\underbrace{\left(1 - kp_{t,T}\frac{L_t}{A_t^{*c}}\right)}_{\text{risk budget}}\right)\left[\sigma_t^{-1}\sigma_\Phi + \sigma_t^{-1}\sigma_B\right]$$

- We now have a state-dependent allocation to PSP versus LHP, and we only recover the static LDI strategy for *k=0*.

- The dynamic strategy is somewhat reminiscent of CPPI strategy, extended to an ALM context, except for $p_{t,T}$ term.

- Dollar allocation to PSP is: $$A_t^{*c} w_t^{PSP} = \underbrace{\frac{\sigma_t^{-1}\lambda}{\gamma}}_{\text{multiplier}}\underbrace{\left(A_t - \underbrace{kp_{t,T}L_t}_{\text{floor}}\right)}_{\text{cushion}}$$

State-Dependent Risk Budget

- The number *p(t,T)* is the *(risk-neutral)* probability that the unconstrained strategy would end up violating the budget constraint, hence between 0 and 1.

- Implications:
  - Risk budget $A(t)-p(t,T)kL(t) > A(t)-kL(t)$ looks bigger than investor's wishes.
  - Yet we have that $A(T)\geq kL(T)$ (in fact $A(t)\geq kL(t)$ for all *t*) because *p(t,T)* adjusts itself in a dynamic manner.

- State-dependent behavior of risk budget:
  - $p(t,T)k \rightarrow k$ at all time *t* when "things go really wrong"
  - $p(t,T)k \rightarrow 0$ at all time *t* when "things go really right"

Another (Equivalent) Interpretation

- Dynamic asset allocation models with a risk-control focus lead to the following optimal terminal wealth level:

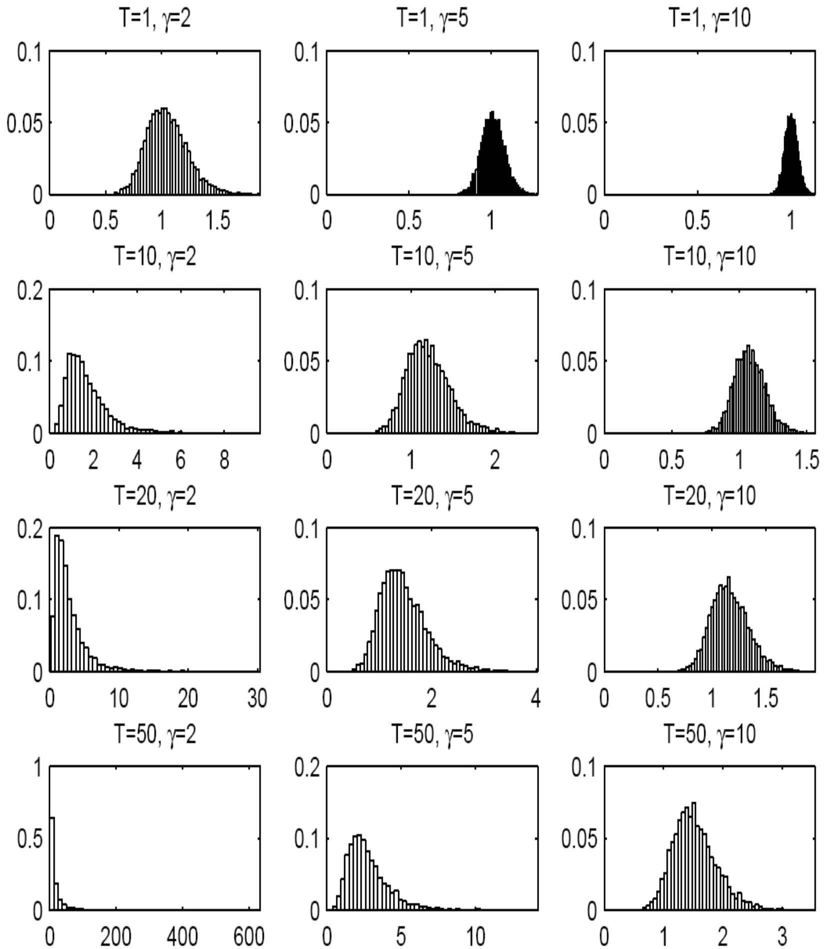
$$\underset{w_t}{Max}\, E\left[u\left(A_T\right)\right] \text{ s.t. } A_t \geq kL_t$$

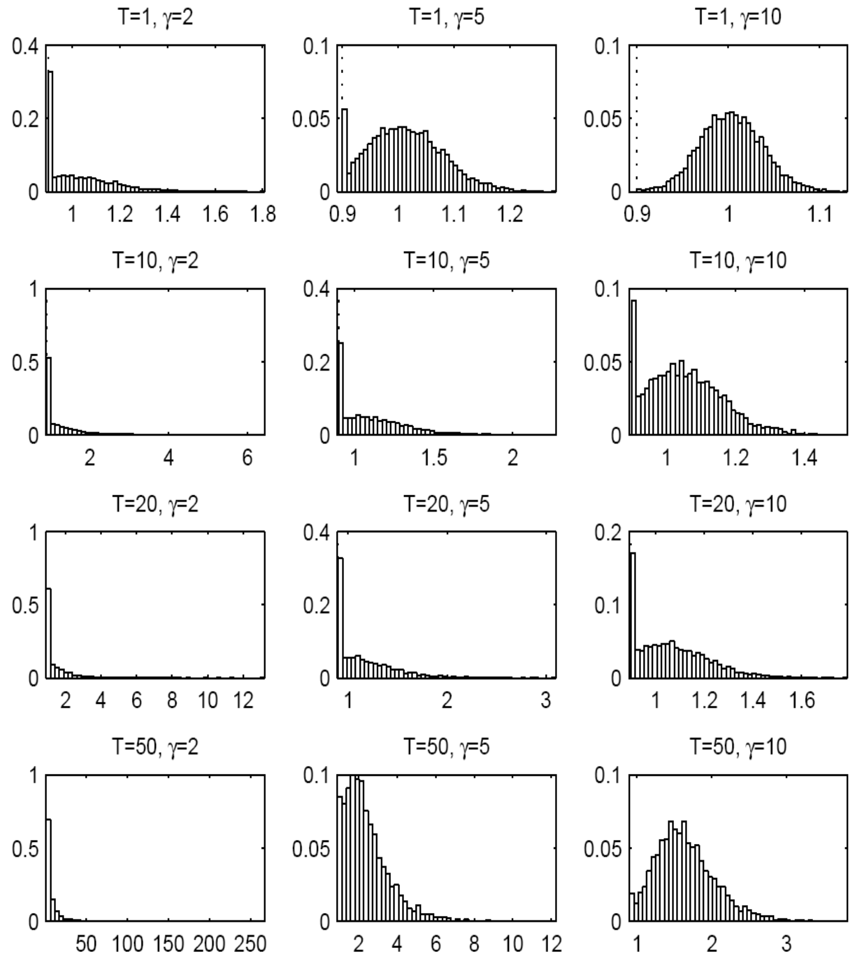
$$\Rightarrow A_T^{*c} = kL_T + \max\left(\xi_T A_T^{*u} - kL_T, 0\right)$$

Wealth generated by *unconstrained* LDI strategy

- Intuition is relatively straightforward.
  - When risk constraints are introduced, investors first aim at investing in a “floor replicating portfolio”.
  - Then they seek maximum upside potential of optimal wealth level A^{*u} achieved under optimal unconstrained LDI strategy.
  - This is now somewhat reminiscent of (dynamic replication of) extended OBPI strategies, with LDI portfolio strategy as “underlying asset”.

Unconstrained vs. Constrained (k=90%)





- Introduction
- From Static to Dynamic LDI Strategies
- Pro-Cyclicality of Risk-Controlled Strategies
- Conclusions and Suggestions for Further Research

Procyclicality of Risk-Controlled Strategies

- Basic risk-controlled strategies like CPPI have often been criticized for their pro-cyclical nature.

- They recommend selling the performance-seeking assets of the downside, and buying them on the upside.
  - This raises the question of equilibrium implications of risk-controlled strategies.
  - It also, and perhaps more importantly, raises the question of the opportunity costs associated with selling on the downside in the presence of mean-reverting equity returns.

- Another (somewhat related) outstanding question is how to minimize the cost of downside protection.

Introducing Mean-Reverting Returns

- Let us introduce a time-varying risk-premium, with a mean-reverting component (Kim and Omberg (1996)).

$$dS_t = S_t\left[r + \sigma_S \lambda_t^s\right]dt + S_t \sigma_S dW_t^S$$
$$d\lambda_t^s = a_\lambda\left(\bar{\lambda} - \lambda_t^s\right)dt + \sigma_\lambda dW_t^\lambda$$

with a negative correlation $\rho_{\lambda S}$ between λ^S and S.

- Unconstrained ALM model can be extended to generate:
  - The investor with $\gamma>1$ holds more stock when the stock Sharpe ratio is mean-reverting than when it is constant ($\sigma_\lambda=0$): equity serves as a hedge against equity holdings;
  - The investment in stock decreases when approaching horizon *T (consistent with TDF prescription, albeit not with their simplistic deterministic implementation);*
  - Investment in stock increases with increases in risk-premium following a significant drop in equity markets.

Mean-Reversion and MFR Constraints

- The following general expression holds for dynamic allocation to the PSP in the constrained vs. unconstrained strategy, even in the presence of a stochastic opportunity set:

$$w_t^{*c} = w_t^{*u} \times \underbrace{\left(1 - p_{t,T}\frac{kL_t}{A_t^{*c}}\right)}_{\text{risk budget}}$$

- If risk premium constant, unconstrained strategy is static LDI.

- On the other hand, assuming mean-reversion in equity returns, fall of equity markets leads to two competing forces:
  - Risk-control aspect: risk budget $1\text{-}p_{t,T}kL_t/A_t$ has decreased, and allocation to equities should be decreased accordingly;
  - Tactical aspect: estimated probabilities of an increase in equity returns have increased, and unconstrained allocation to equities (PSP) w^u_t should be increased accordingly.
  - Depending on parameter values, one effect dominates the other, with risk management always prevailing ultimately.

Revisiting Pro-Cyclicality

- There is another reason why a fall in equity prices should not always lead to a decrease of equity allocation, even without mean-reverting equity returns.

- Introduce utility satiation, capturing preference asymmetry:

$$\underset{w_s,\, t\leq s\leq T}{Max}\ E_t\left[u\left({}^{A_T}\!\big/_{L_T}\right)\right] \text{such that } k \leq {}^{A_t}\!\big/_{L_t} \leq k' \text{ a.s.}$$

- Overall, given the complexity of surplus sharing rules, it is unclear whether pension funds have any utility over exceedingly large surpluses.

Introducing an Upper Bound – So What?

- Optimal terminal asset value (with the constant ξ‘ chosen so that the budget constraint holds):

$$A_T^{*k,k'} = kL_T + \max\left(\xi' A_T^{*u} - kL_T, 0\right) - \max\left(\xi' A_T^{*u} - k' L_T, 0\right)$$

- The optimal strategy consists of a long position in an exchange “option” and a short position in an exchange “option” on the same underlying payoff and a higher strike price (bull spread option).

- The idea is that by giving up part of the upside potential beyond levels where marginal utility of wealth (relative to liabilities) is low or almost zero, the investor can decrease the cost of downside protection: $\xi' > \xi$

- Increases/decreases in equity prices lead to different effects depending on whether one is closer to the floor vs. goal.

- Introduction
- From Static to Dynamic LDI Strategies
- Pro-Cyclicality of Risk-Controlled Strategies
- Conclusions and Suggestions for Further Research

Conclusion & Extensions

- In a dynamic setting, we obtain analytical expressions for the optimal investment policy when pension payments are subject to inflation and interest rate risks.

- In the presence of MFR constraints, the optimal policy is shown to involve dynamic trading strategies that are reminiscent of CPPI or OBPI portfolio insurance strategies, extended to a relative risk context.

- The introduction of maximum funding ratio targets would allow pension funds to decrease the cost of downside protection.

- As an extension, it would be useful to develop an integrated ALM model, taking into account the perspective of the shareholder of the (financially constrained) sponsor company.