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This chair, led by Lionel Martellini, Scientific Director of EDHEC-Risk Institute, is examining dynamic allocation strategies in asset-liability management in order to formulate an integrated ALM model.

The current paper is an attempt to analyse the valuation of pension liabilities, regarded as defaultable claims issued by the sponsor company to workers and pensioners, in the context of a structural intertemporal capital structure model with contingent contributions. Our model has important policy implications in that it provides a first step towards a much needed methodological framework for the design of firm-specific regulatory constraints and accounting valuation principles. It also has a number of implications in terms of investment decisions at the pension fund level, and funding decisions at the sponsor company level.

I would like to thank the co-authors, Lionel Martellini and Vincent Milhau, for the quality of their analysis and research. We hope that you will find the paper valuable and will continue to monitor and contribute to our research in this area.

We would also like to extend our warm thanks to our partners at BNP Paribas Investment Partners for their collaboration on the project and their commitment to this research chair.

Wishing you a pleasant and informative read,

Noël Amenc
Professor of Finance
Director of EDHEC-Risk Institute
About the Authors

Lionel Martellini is professor of finance at EDHEC Business School and scientific director of EDHEC-Risk Institute. He has graduate degrees in economics, statistics, and mathematics, as well as a PhD in finance from the University of California at Berkeley. Lionel is a member of the editorial board of the Journal of Portfolio Management and the Journal of Alternative Investments. An expert in quantitative asset management and derivatives valuation, Lionel has published widely in academic and practitioner journals and has co-authored textbooks on alternative investment strategies and fixed-income securities.

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Abstract
Correctly assessing the value of a pension plan in deficit with a weak sponsor company is a real challenge given that no comprehensive model is currently available for the joint quantitative analysis of capital structure choices, pension fund allocation decisions and their impact on rational pricing of liability streams. This paper is an attempt to fill this gap by analyzing the valuation of pension liabilities regarded as defaultable claims issued by the sponsor company to workers and pensioners in the context of an integrated model of capital structure. Our results show that leverage decisions have a strong impact on the fair value of pension liabilities, and conversely that the presence of a pension plan decreases the optimal leverage ratio. We also find that interior optimal values may exist for allocation decisions. In an extension to a dynamic setting we find that risk-controlled strategies allow the pension fund to take more risks, which has a positive effect on equity value, while protecting pensioners. Our model has important policy implications in that it provides a first step towards a much needed methodological framework for the design of firm-specific regulatory constraints and accounting valuation principles.
Executive Summary
One of the main risks for pension plan participants, indeed the only source of risk in a defined-benefit plan with unconditional liability payments, is that of sponsor bankruptcy when the pension plan is underfunded. In an attempt to address this risk, the legal, accounting, and fiscal environments of corporate pension funds have undergone great changes; these changes have led to closer scrutiny of the valuation of pension liabilities and of the impact of both the market and credit risk components on the value of pension obligations. Correctly assessing the value of a pension plan in deficit with a weak sponsor is nonetheless a great challenge, as there is no comprehensive model for the joint quantitative analysis of capital structure choices, pension fund allocation decisions, and their impact on rational pricing of liability streams. In fact, international accounting standards SFAS 87 and IAS19 recommend that pension obligations be valued at a discount rate equal to the market yield on AA corporate bonds, the same rate for all firms. Although using a market rate is arguably an improvement on using a constant rate (including a credit spread component or not) independently of market conditions, the use of the same market rate to discount all pension liabilities regardless of the sponsor’s credit rating, pension funding situation, and asset allocation policy is unlikely to lead to an accurate assessment by stakeholders of the impact of specific default risk on the value of pension obligations. This paper attempts to fill this gap by analyzing the valuation of pension liabilities regarded as defaultable claims issued by the sponsor to workers and pensioners in the context of an integrated model of capital structure.

It focuses on the interaction between allocation decisions of the pension plan and the valuation of these liabilities, thereby extending the literature on capital structure and that on defaultable bond pricing to account for the presence of a pension plan.

Our model is a stylized representation of the relationships between the stakeholders of a company with a pension plan, including shareholders of the sponsor company, bondholders, and beneficiaries of the pension fund (workers and pensioners). The model can be summarized as follows. The sponsor issues a debt with face value $D$, and also issues pension claims, perceived as a collateralized form of debt held by workers and pensioners, with face value $L$. The initial capital of the firm is allocated to funding investment projects (company asset value denoted by $V$) and to funding the pension plan (pension asset value denoted by $A$). The pension fund allocates a fraction $\omega$ of the initial endowment to some performance-seeking portfolio (PSP) and a fraction $1-\omega$ to some liability-hedging portfolio (LHP). If the assets of the pension fund $A$ are insufficient to deliver the promised pension payment $L$, the sponsor makes a contribution equal to the deficit $L - A$. If the sponsor is unable to make this contribution, default is triggered. If debt cannot be fully repaid, bankruptcy is also triggered. When default has not been triggered, equity-holders receive a fraction of this surplus, which can be used to pay back bondholders. If debt cannot be fully repaid, bankruptcy is also triggered. When default has not been triggered, equity-holders are left with the remaining assets of the pension fund and the sponsor, plus their access to surpluses. Otherwise, they receive nothing. We also incorporate tax effects, bankruptcy costs,
and contributions triggered by the presence of regulatory funding ratio constraints (see exhibit 1).

Under standard assumptions regarding the dynamics followed by all variables of interest, including the return on the performance-seeking portfolio and the return on the real assets held by the firm, one can use option-pricing theory to find the rational value of the claims held by all stakeholders, and also analyze the impact on the value of these claims of funding and leverage decisions made by the sponsor, as well as asset allocation decisions made by the pension fund. The main ingredients of the model are the size of the pension fund relative to the assets of the sponsor (L/V), the size of the pension assets relative to the pension liabilities (the funding ratio A/L), and the size of the outstanding debt relative to the assets of the sponsor company (D/V). Other important parameters are those defining the allocation strategy of the pension fund, as well as the correlation between the return on pension assets and the return on sponsor assets (see exhibit 2). ¹

Our findings have two main implications, macro (with a number of possible policy recommendations for pension fund regulators), and micro (with a number of strategy recommendations for pension fund managers).
1. Macro/policy implications

Our results have important potential policy implications. In particular, they raise a series of questions related to current practices in terms of liability cash-flow valuation, in which accounting regulations recommend an arbitrary market rate identical for all firms. They also raise questions about prudential regulation, as they provide new formal insights into the impact of the presence of short-term funding ratio constraints or the presence of pension insurance.

(a) Accounting regulations

Our model suggests that valuation principles for liability streams should account for differences in financial health and capital structure decisions made by the sponsor, as well as differences in asset allocation policy made by the pension fund. Our analysis also suggests that any minimum funding requirements should be a function of these items as well because they are based on estimates of fair pension liability value. We first analyze the structural relationship between the value of the assets of the firm and the value of the claims issued by the sponsor to bondholders and liability-holders respectively, in the context of a formal capital structure model. Explicitly recognizing that bondholders and liability-holders hold (relatively complex) payoffs that are functions of both pension fund and firm assets allows us to obtain a wealth of new insight into the interaction of capital structure decisions and the valuation of pension claims, as well as into the interaction of asset allocation decisions and the sponsor’s credit rating. We show that the existence of a pension plan has a strong impact on capital structure decisions, with the optimal leverage ratio a decreasing function of promised pension payments. This finding is consistent with the intuition that suggests that a firm without a pension fund would optimally take on more debt than an otherwise identical firm sponsoring a pension plan, since the latter firm has already issued a form of debt by committing to a payment to retired employees. We also find that the presence of a pension fund has a substantial impact on debt value and credit ratings (see exhibit 3).

These results suggest that a fair assessment of a firm’s creditworthiness can be done only when analysts and rating agencies have an integrated view of the firm’s financial situation, a view that also takes into account the pension fund’s situation and its funding status as well as its allocation strategy. Conversely, capital structure decisions have a substantial impact on the fair value of pension claims; the creditworthiness of liabilities is a decreasing function of the leverage ratio (see exhibit 4).

One important corollary is that the regulatory valuation leads to overestimates of the fair value of liabilities for highly leveraged firms, whereas it leads to underestimates for firms with little debt outstanding, and one of the contributions of the model is to enable a quantitative estimate of the magnitude of the over- or underestimate as a function of parameter values. On the whole, our research has important policy implications in that it takes a first step towards a much-needed methodological framework for the design of firm-specific regulatory constraints and liability valuation principles; it also calls for the emergence of a scheme-specific rule for pricing pension insurance. Although providing firm-specific regulatory constraints and
accounting valuation principles might seem a formidable challenge, our paper presents an analytical model that could be used to achieve such objectives. It should be emphasized that a move towards an endogenous discount rate rationally taking into account credit risk in liability streams would lead to counter-cyclical regulation, in the sense that sponsors in bad shape will face a loosening of the funding ratio constraints since a pension fund will report an improvement in the surplus or funding...
ratio in the event of a decline in the credit quality of pension liabilities. It can be argued that reporting a gain from a decline in credit quality is potentially misleading and can mask a deteriorating situation. On the other hand, it can also be argued that a loosening of the funding ratio constraints in difficult situations might help maintain the sustainability of the defined-benefit pension system. Of course, in a rational expectation model, employees of the sponsor would observe the deterioration in the present value of their pension benefits and bargain for increases in wages to compensate for the loss, an option that would not be available to pensioners.

(b) Prudential regulations
In this paper, we extend the analysis of funding and investment decisions at the pension fund, and their interaction with leverage and contribution decisions made by the sponsor, to an intertemporal setting; previous literature has, for the most part, considered simple static models. Extending the analysis to the dynamic setting is of critical relevance, because it allows us to analyze a number of empirically relevant features such as the presence of short-term minimum funding ratio constraints imposed by regulators in most developed countries (with the notable exception of the United States and the United Kingdom). Such short-term funding constraints, whose presence cannot be accounted for in a static setting, have a great impact on the valuation of pension liabilities, and on shareholder and total firm value, by forcing the sponsor to make early additional contributions when the poor performance of the pension fund portfolio makes such additional contributions necessary (see exhibit 5). As expected, we find that pensioners benefit unambiguously from the enforcement of minimum funding requirements, with the highest benefits to be expected when the initial funding ratio is low, which is when pension protection is most critically needed. The situation is more ambiguous for equity-holders. On the one hand, it can be argued that early contributions make insolvency of the pension fund less likely. On the other hand, they may also prove ex post to have been unnecessary.

Exhibit 5: Impact of short-term constraints on stakeholders’ welfare.
These figures perform comparative static analysis with respect to the fraction of surpluses that is attributed to equity holders when the pension plan is terminated. The firm is assumed to be positively correlated with the market, and three values of the initial regulatory funding ratio are considered (70%, 100% and 130%). Parameters are fixed at their base case values (see table 1 page 95).
if strong stock market performance can enable the fund to recover without calling for exceptional contributions. On the whole, we find that equity-holders will benefit from the introduction of short-term constraints only if their access to the pension fund surpluses is sufficiently high, in which case they can expect unnecessary contributions to be returned to them at the terminal date. In any case, the marginal cost of introducing short-term constraints is relatively modest for equity-holders, which suggests that such prudential regulations could be a welcome ingredient in pension fund management.3

2. Micro/strategy implications
We also obtain novel results regarding optimal funding and investment decisions.

(a) Investment decisions
When the correlation of the value of the firm process and the stock index return process is positive, we find that the fair value of promised payments to bondholders and pensioners is a decreasing function of the allocation to risky assets by the pension fund. This is a clear case of asset substitution, since a higher allocation to risky assets leads to an increase in the total riskiness of the total assets held by the firm (financial assets held off the balance sheet through the pension fund and real assets held directly on the balance sheet), which is the underlying state variable on which is based the value of such claims. When the correlation is negative, however, greater allocation to risky assets may induce diversification benefits. This competition between the asset substitution effect and the diversification effect, which has never been analyzed in the related literature, leads to an interior optimal solution to the problem of maximizing total firm value (and to the problem of maximizing pensioner value), at least for reasonably low funding ratios. On the whole, there is, in general, clear evidence of conflicts of interest between the stakeholders, between shareholders and pensioners, in particular. If it is assumed that they do not have access to any pension fund surplus, risk taking is detrimental from the pensioners’ perspective, because it involves increasing the likelihood of partial recovery of pension claims, whereas the upside potential of the performance-seeking assets allows shareholders to reduce the burden of contributions needed to meet expected pension payments [see exhibit 6].

These conflicts of interest could be mitigated by granting pensioners partial access to the surplus (see conditional indexation rules in the Netherlands), thereby allowing plan beneficiaries to benefit from the expected performance of more aggressive investment strategies. More generally, our results have implications in terms of the optimal design of pension plans, since they advocate the emergence of more subtle surplus sharing rules, which could include, say, hybrid retirement plans, contribution holidays for defined-benefit plans, or both, which would allow equity-holders to reduce the contribution burden, all while protecting the interests of pensioners. We also find that an effective way to align the interests of shareholders and pensioners without any complex adjustment to the pension plan structure is to enlarge the set of admissible investment strategies so as to include dynamic risk-controlled strategies such as constant proportion portfolio
insurance (CPPI), or their extension to a pension management context sometimes referred to as contingent immunization or dynamic liability-driven investment (LDI). In fact, implementing risk-controlled strategies that attempt to ensure a funding ratio above 100% allows shareholders (limited) access to the expected upside of risky assets, while ensuring that pensioners will not be hurt by the increase in risk (see exhibit 7).

(b) Funding decisions
We first find that pensioners always benefit from increases in funding to the pension plan because the plan assets are used as collateral for the pension claims. This increase in the value of liabilities takes place at a decreasing rate: when the pension plan is already very well funded, marginal increases in asset value generate only marginal improvements in pensioner welfare. The impact on equity-holders is, in principle, positive as well, at least when shareholders have full access to the pension fund surpluses and when the initial funding ratio is sufficiently high:

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Exhibit 6: Impact of allocation decisions on stakeholder welfare.
These figures perform comparative static analysis with respect to the allocation to the risky asset, when the firm is positively correlated with the market. The pension fund is fully funded in the regulatory sense at the initial date. Parameters are fixed at their base-case values (see table 1 page 95).

Exhibit 7: Impact of allocation decisions with risk-controlled strategies.
These figures perform comparative static analysis with respect to the multiplier of the constant-proportion portfolio insurance (CPPI) strategy when the firm is positively correlated with the market. The initial funding ratio is 130%, so as to create a positive risk budget. Parameters are fixed at their base-case values (see table 1 page 95). The vertical line identifies the base case, where the initial weight allocated to the stock is 50%.
increases in funding lead to increases in tax benefits, and therefore to higher shareholder value; when shareholders have full access to pension surpluses, the tax benefits come with no opportunity costs. For large pension claims, an increase in funding starting from low funding ratios can, on the other hand, be detrimental to shareholders: when it is highly unlikely that pension assets will ever be in excess of pension liabilities, one additional dollar invested in the pension fund is certain to be lost for shareholders, who would sooner have it invested in the firm. We also analyze the value of the bonds as a function of the allocation to the pension fund. As one would have expected, a firm with a pension fund in surplus has a better rating quality than a firm with a pension fund in deficit, all else equal, and the impact of pension funding decisions is quite substantial. These results are consistent with empirical findings that have documented the positive influence of pension funding on credit ratings. Moreover, we find that the introduction of a pension benefit guarantee corporation would have a great impact on optimal funding decisions. We also analyze the determinants of the difference between the fair and regulatory values of pension insurance, which can be substantial for reasonable parameter values.

Our work can be extended in a number of directions. First, the model could usefully be extended to account for the indexation, in most countries, of promised pension payments to price or wage inflation, in accordance with pre-specified rules. An analysis of the impact of (conditional) inflation indexation on the value of stakeholder claims would therefore be a worthwhile extension of this paper. In terms of allocation strategies, we have tested in this paper fixed-mix strategies as well as basic risk-controlled strategies and found that the benefits of moving away from static allocation strategies to even the simplest form of dynamic risk-controlled strategies were substantial for both shareholders and pensioners. It would be useful to try to test more sophisticated forms of welfare-improving strategies in a more general dynamic context, including strategies with a floor given as a function of the (regulatory and/or fair) value of the liability portfolio, strategies with a performance cap in addition to floors, which can allow for a decrease in the cost of downside risk protection, as well as strategies involving corporate bonds in the liability-hedging portfolio. Moreover, one would also like to test risk-controlled strategies that encompass state variables related to the sponsor. The intuition is that one could further decrease the cost of downside protection by relaxing risk constraints when the sponsor is financially healthy and thus make up for eventual deficits through increased contributions, while focusing on hedging away the states of the world characterized by a joint occurrence of poorly performing pension assets and poor financial health on the sponsor’s part.
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In recent decades, a number of great changes have taken place in the legal, regulatory, accounting, and fiscal environments of corporate pension funds; collectively, these changes have led to significantly heightened scrutiny of the valuation of pension liabilities, with a focus on greater transparency with respect to the impact of both market and credit risk components on pension obligation values. In the US, the most substantial change in the pension environment was the passage of the Employee Retirement Income Security Act (ERISA) in 1974, which has led, among other things, to corporate pension liabilities becoming parts of corporate liabilities. While a significant evolution, implying that beneficiaries could use firm assets to cover any deficit with respect to accrued liabilities in the event of termination of a pension plan, the impact of integrating pension liabilities in sponsor companies’ balance sheets and income statements was somewhat softened by the fact that pension liabilities were reported at their historical value. As a result, the impact of changes in market and credit risk factors on the value of the pension obligation could not be accurately assessed by beneficiaries and by shareholders of the sponsor. More recently, the move towards fair valuation in global accounting standards has generated renewed interest in a more dynamic measurement of pension liabilities, with a focus on the impact not only of market risk (interest rate and inflation risks) but also of credit risk. In 2006, the FASB issued SFAS 157, which states (in paragraph 15): “The reporting entity shall consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value.” Obviously, this argument makes compelling sense from a financial economics standpoint. Its implications in the specific context of pension liability valuation, however, are yet to be fully recognized. In fact, international accounting standards SFAS 87.44 and IAS19.78 recommend that pension obligations be valued on the basis of a discount rate equal to the market yield on AA corporate bonds, the same rate for all firms. Although using a market rate is arguably an improvement on using a constant rate (including a credit spread component or not) independently of market conditions, using the same market rate to discount all pension liabilities, regardless of the sponsor’s credit rating, pension funding situations, and asset allocation policy is unlikely to lead to accurate stakeholder assessments of the impact of specific default risk on the value of pension obligations. This is a severe problem, as can perhaps best be evidenced by the failure of pension fund regulations (both accounting and prudential) in developed countries to protect beneficiaries from events combining pension fund underfunding and sponsor default. One of the main risks for plan participants, actually the only source of uncertainty for a defined-benefit plan with unconditional liability payments, is precisely that of sponsor bankruptcy when the pension plan is underfunded. Although this risk is somewhat mitigated by the presence of a nation-wide pension guarantee fund such as the Pension Benefit Guaranty Corporation in the US or the Pension Protection Fund in the UK, benefits usually fall sharply when transferred to such a guarantee fund. For example, when United Airlines filed for bankruptcy (Chapter 11), a controversial filing that allowed it to cancel its pension obligations and transfer.
these obligations to the Pension Benefit Guaranty Corporation (PBGC), United Airlines employees lost $3.2Bn because pension benefits are not fully insured by the PBGC. This loss meant anywhere from 20% to more than 50% of pension rights (Amenc, Martellini, and Sender 2009). One additional concern is the presence of systemic risk for the pension fund industry if many plan sponsors simultaneously default on their pension obligations. Regulatory bodies in charge of pension fund supervision, as well as pension fund beneficiaries and pension fund managers, thus have great incentives to monitor changes in the joint probability distribution of default risk from the sponsor and underfunding at the pension fund, as well as to assess the implications in terms of changes in the fair value of liability obligations.

Correctly assessing the value of a pension plan in deficit with a weak sponsor is, however, a real challenge. Although a large body of literature on pension economics has generated a wealth of useful qualitative insight, there is no realistic comprehensive asset/liability management model for the joint quantitative analysis of capital structure choices, pension fund allocation decisions, and the rational pricing of liability streams. The development of such a model, and its application to an empirical analysis of the mispricing of liability streams, is, as it happens, the focus of our paper. In this context, we contribute to the literature on pension economics by analyzing the valuation of pension liabilities regarded as defaultable claims issued by the sponsor to workers and pensioners in the context of a structural intertemporal capital structure model with contingent contributions. Our paper focuses on the interaction of allocation decisions of the pension plan and the valuation of these liabilities, thereby extending the literatures on capital structure literature (e.g., Leland (1994) and Leland and Toft (1996)) and defaultable bond pricing (Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995) and many other subsequent papers) to account for the presence of a pension plan.

On the one hand, several relatively recent papers have addressed optimal portfolio decisions in the presence of liability constraints, including Merton (1993), Sundaresan and Zapatero (1997), Rudolf and Ziemba (2004), van Binsbergen and Brandt (2007) or Martellini and Milhau (2008). For the most part, however, these papers have considered the pension fund problem in isolation, without taking into account the relationship with the sponsor. As a result, the liability value process is exogenously specified in all these as well as other related papers. This is a serious limitation since it fails to recognize that pension liabilities are a particular claim on the assets of the firms, and, more precisely, that they are a collateralized form of debt owned by the workers and pensioners of the sponsor, where the assets of the pension plans are the collateral, in exchange for which the company receives the present value of lower wage demands. In particular, what is missing in such analyses is the taking into account of credit risk in liability streams, and how it is affected by capital structure decisions at the sponsor and by asset allocation decisions at the pension fund. On the other hand, a series of early papers has looked at the pension fund problem from a corporate finance perspective (see early papers by Sharpe (1976), Treynor (1977), Black (1980), Tepper (1981), Harrison and Sharpe (1983), Bicksler and Chen (1985)).

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Those papers, however, were cast in a highly stylized one-period model (one notable exception being Harrison and Sharpe (1983)), with constant interest rates, and in the absence of proper modeling of intermediate contributions from the sponsor to the pension plan and with no formal analysis of capital structure decisions, asset allocation decisions and the rational value of liability streams. As a result of the highly stylized nature of those early models, the predictions they generate cannot be used to properly assess the value of default risk in pension obligations from a purely quantitative standpoint. In fact, most of those papers produce only extremal solutions to the optimal pension funding and investment policy problems involving either funding as little as possible and using allocation decisions to maximize default risk so as to take advantage of the insurance provided by the nation-wide pension guarantee fund if it exists (Sharpe 1976), or funding to the greatest extent and investing fully in safe liability-matching assets to capture the preferential treatment of pension plans under current tax law (see Black (1980) and Tepper (1981)). In a related effort, Harrison and Sharpe (1983) also obtain corner solutions in pension funding and investment strategies while simultaneously taking into account the tax and insurance effects, depending on whether the insurance or the tax effect dominates. One exception is Bicksler and Chen (1985), arguably the paper most closely related to ours, who show that interior solutions could exist depending upon the relative strengths of the tax and the insurance effects in the presence of frictions such as pension termination costs and progressive and asymmetric corporate income tax structure. We extend this paper and the related literature in a number of important dimensions.

We first explicitly analyze the structural relationship between the value of the assets of the firm and the value of the claims issued by the sponsor to bondholders and liability-holders respectively, in the context of a formal capital structure model. Explicitly recognizing that bondholders and liability-holders hold (relatively complex) payoffs that are functions of the pension fund assets but also the firm assets allows us to obtain a wealth of new insight into the interaction of capital structure decisions and the valuation of pension claims, as well as into the interaction of asset allocation decisions and the sponsor’s credit rating. We first show that the existence of a pension plan has a strong impact on capital structure decisions, with the optimal leverage ratio a decreasing function of promised pension payments. This finding is consistent with the intuition that suggests that a firm without a pension fund would optimally take on more debt than an otherwise identical firm sponsoring a pension plan, since the latter firm has already issued a form of debt by committing to a payment to retired employees. Conversely, capital structure decisions have a substantial impact on the fair value of pension claims, with a pension credit spread that increases approximately proportionally to the leverage ratio. One important corollary is that the regulatory valuation leads to overestimates of the fair value of liabilities for highly leveraged firms, whereas it leads to underestimates of the liability for firms with little debt outstanding, and one of the contributions of the model is to enable a quantitative

6 - See Hu (1992) for an integrated analysis of pension funding and corporate financing strategies in the presence of default risk, albeit in a static setting.
estimate of the magnitude of the over- or underestimate as a function of parameter values. We also obtain novel results regarding optimal funding and investment decisions. In particular, we obtain interior solutions for some parameter values even in the absence of pension termination costs and progressive and asymmetric corporate income tax structures. More specifically, when the correlation of the value of the firm process and the stock index return process is positive, we find that the fair value of promised payments to bondholders and pensioners decreases following an increase in the allocation to risky assets by the pension fund. This is a clear case of asset substitution, since a higher allocation to risky assets leads to an increase in the total riskiness of the total assets held by the firm (financial assets held off the balance sheet through the pension funds and real assets directly held on the balance sheet), which is the underlying state variable on which is based the value of such claims. When the correlation is negative, on the other hand, a higher allocation to risky assets may induce diversification benefits. This competition between the asset-substitution effect and the diversification effect, which has not been analyzed in previous papers, leads (at least for reasonably low funding ratios) to an interior optimal solution to the pension investment strategy problem, both when the objective is related to maximizing total firm value and when it is related to maximizing pensioner value. On the whole, there is, in general, clear evidence of conflicts of interest between stakeholders, between shareholders and pensioners, in particular. From the pensioners’ perspective, risk taking is detrimental because it involves increasing the likelihood of partial recovery of pension claims. On the other hand, the upside potential of performance-seeking assets allows shareholders to reduce the burden of contributions needed to meet expected pension payments. These conflicts of interest could be mitigated by granting pensioners partial access to the surplus (conditional indexation rules in the Netherlands), thereby allowing plan beneficiaries to benefit from the increase in expected performance implied by more aggressive investment strategies. We also find that another way to align the interests of shareholders and pensioners is to enlarge the set of admissible investment strategies so as to include dynamic risk-controlled strategies such as constant proportion portfolio insurance (CPPI) (Black and Jones (1987) and Black and Perold (1992)), or their extension to a pension management context sometimes referred to as contingent immunization (Bodie 1990a) or dynamic liability-driven investment (LDI) (Martellini and Milhau (2008)). In fact, we demonstrate that risk-controlled strategies that attempt to ensure a funding ratio above 100% allow shareholders (limited) access to the upside performance of risky assets, while ensuring that pensioners will not be hurt by the induced increase in risk. In terms of funding decisions, we first find that pensioners always benefit from increases in funding to the pension plan because the plan assets are used as collateral for the pension claims. This increase in the value of liabilities takes place at a decreasing rate: when the pension plan is already very well funded, marginal increases in asset value generate only marginal improvement in pensioner welfare. The impact on equity-holders is, in principle, positive as well, at least when shareholders have full access to

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the pension fund surpluses and when the initial funding ratio is sufficiently high: increases in funding lead to increases in benefits from the tax advantage and therefore to higher shareholder value; when shareholders have full access to pension surpluses, the tax benefits come with no opportunity costs, which would be a factor in the event of partial or zero access by shareholders to pension surpluses. For large pension claims, an increase in funding starting from low funding levels can, on the other hand, be detrimental to shareholders: when it is highly unlikely that pension assets will ever be in excess of pension liabilities, one additional dollar invested in the pension fund is certain to be lost for shareholders, who would sooner have it invested in the firm. We also analyze the value of the bonds as a function of the allocation to the pension fund. As one would have expected, a firm with a pension fund in surplus has a better rating quality than a firm with a pension fund in deficit, all else equal, and the impact of pension funding decisions is found to be quite substantial. These results are consistent with the empirical findings by Martin and Henderson (1983) or Carroll and Niehaus (1998), who show that pension funding has a positive influence on credit ratings. In addition, the introduction of a pension benefit guarantee corporation would have a great impact on optimal funding decisions. In our model, maximizing the total value in the presence of the PBGC with respect to corporate and pension fund policies and maximizing the difference between the fair and the regulatory values of the pension put are not equivalent objectives because of the presence of frictions (tax shields and bankruptcy costs) that were not present in Sharpe (1976), for example. For large promised payments to pensioners, and given our base-case parameter values, we obtain that funding the pension plan to the maximum is the optimal solution from a perspective of maximizing firm value. Finally, we also analyze the determinants of the difference between the fair and regulatory value of pension insurance, which can be substantial for reasonable parameter values.

From a methodological standpoint, one additional contribution of the paper is to extend the analysis of funding and investment decisions at the pension fund, and their interaction with leverage and contribution decisions at the sponsor, to an intertemporal setting while Bicksler and Chen (1985) and most related papers have considered only a simple static model. Extending the analysis to the dynamic setting turned out to be of critical relevance, because it allows us to analyze a number of empirically relevant features such as state-dependent contribution policies, or the presence of short-term minimum funding ratio constraints imposed by regulators in most developed countries (with the notable exception of the US and the UK). Such short-term funding constraints, whose presence cannot be accounted for in a static setting, have a dramatic impact on the valuation of pension liabilities, as well as on shareholder and total firm value, by forcing the sponsor to make early additional contributions when the poor performance of the pension fund portfolio makes such additional contributions necessary. Unsurprisingly, we find that pensioners benefit unambiguously from the enforcement of minimum funding requirements, with the highest benefits to be expected when the initial funding ratio is low, which is when pension protection

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1. Introduction

is most critically needed. The situation is more ambiguous for equity-holders. On the one hand, it can be argued that early contributions make insolvency of the pension fund less likely. On the other hand, they may also prove, after the fact, to have been unnecessary if strong stock market performance enables the fund to recover without calling for exceptional contributions. On the whole, we find that equity-holders will benefit from the introduction of short-term constraints only if their access to pension fund surpluses is sufficiently high, in which case they can expect unnecessary contributions to be returned to them at the terminal date.

In addition to these contributions, when addressing the question of pricing equity of sponsors with underfunded plans, we also shed new light on the empirical findings in Franzoni and Marin (2006), who document systematic mispricing of US companies sponsoring defined-benefit (DB) pension plans (see also Picconi (2006), who conclude that even analysts appear to misinterpret readily available information about firms’ pension earnings and funding status). In other words, although previous empirical research documented the presence of a relationship between pension fund funding status on the one hand and equity and debt prices on the other (Jin, Merton, and Bodie (2006) find empirical evidence that equity risk reflects pension risk despite opaque accounting rules and Carroll and Niehaus (1998) find empirical evidence for a positive relation between the funding of DB plans and debt ratings), there is reason to believe that the market incorrectly assesses the price of claims on the firm’s assets in the presence of a pension fund. It is our hope that our paper can contribute to mitigating this mispricing problem. As noted in Rauh (2009), corporations with defined-benefit pension plans in fact face conflicting motives when it comes to managing the uncertainty of the cash flows. On the one hand, the theory of asset substitution (Jensen and Meckling 1976) suggests that increasing the volatility of the assets once the debt is in place increases the value of shareholders’ equity holdings. While a variety of debt covenants can exist that might prevent managers from increasing the volatility of the real assets of the firms, it is important to note that increasing the equity allocation within the pension fund is a straightforward means of increasing the volatility of the financial assets that the firm holds indirectly through the pension fund. Our model allows us to analyze this question through a formal quantitative analysis of the impact on stakeholders (shareholders, bondholders, and pensioners) of an increase in the allocation to equity within the pension plan. On the other hand, reducing the volatility of the assets in place would allow the firm to reduce the likelihood of bankruptcy and the associated costs (Smith and Stulz 1985) and/or reduce the likelihood of being unable to undertake profitable investment projects (Mayers and Smith (1987) and Froot, Scharfstein, and Stein (1993)). Here again, the investment policy at the pension fund can be used to reduce the volatility of the pension fund surplus (by investing in liability-matching instruments), or even by taking into account the presence of the assets of the firm (by shorting financial assets that would be positively correlated with the firm’s real assets). The empirical analysis done by Rauh (2009) suggests that, on average, risk management incentives to avoid costly bankruptcy dominate risk
shifting (asset substitution motives) in pension fund investing. Our paper provides a normative framework for the quantitative analysis of the comparative impact of risk-management and asset-substitution motives on shareholder wealth.

This paper also has important potential policy implications. In particular, it raises a series of questions related to current practices in terms of liability cash-flow valuation (where an arbitrary rate is used) and pension insurance pricing (where the premium paid is independent of both the financial health of the sponsor and pension fund allocation decisions). Our model suggests that valuation principles for liability streams (and derivative assets written on them) should account for differences in financial health and capital structure decisions at the sponsor, as well as differences in asset allocation policy at the pension fund. Our analysis also suggests that any minimum funding requirements should be a function of these items as well.

Although providing firm-specific regulatory constraints and accounting valuation principles might seem a formidable challenge, our paper presents an analytical model that could be used to achieve such objectives. Also somewhat related to ours is a recent paper by Novy-Marx and Rauh (2009), in which the authors provide estimates for the present value of state pension liabilities using both a risk-free discount rate and a discount rate that reflects the state's riskiness in some simplified manner (done by using the yield on the state's general obligation debt).

The rest of the paper is organized as follows. In section 2, we develop the basic intuitions in the context of a simple one-period setting with default triggered only at terminal date, in the spirit of the early Merton (1974) model (referred to as M74 in the paper), and distinguish between the presence and the absence of a benefit guarantee corporation. In section 3, we extend the model to an intertemporal setting, and we analyze the impact of dynamic allocation strategies, a random date of default, short-term funding ratio constraints, and contingent contributions on the value of the various claims. Section 4 concludes and discusses possible directions for further research.
2. A Stylized Model with Default at Terminal Date Only
2. A Stylized Model with Default at Terminal Date Only

In this section, we consider a simple one-period setting with constant interest rates and default triggered only at terminal date so as to develop the main intuitions. This model builds on the model of integrated ALM set up by Scherer (2005).

2.1 The Model

We first consider a situation in which the cost of default is fully borne by pensioners and employees of the sponsor. We subsequently introduce (see subsection 1.3 below) a pension benefit guarantee corporation (PBGC), which is intended to provide pension benefits for pensioners of companies that are in distress.

2.1.1 State Variables

We consider a firm issuing a single class of debt, which we assume to promise a fixed payment \(D\) at time \(T\). The firm has an after-tax unlevered asset value \(V\), which we assume to evolve under the risk-neutral measure \(\mathcal{Q}\) as:

\[
\begin{align*}
    dV_t &= V_t[r\,dt + \sigma_V\,dz^V_t] \\
\end{align*}
\]  

(1.1)

where \(r\) is the constant short-term risk-free rate and \(z^V\) is a standard Wiener process. This dynamics can be modified at minor cost to accommodate for a payout rate \(\delta\) representing all intermediate payments to the agents that hold claims on the firm: it would suffice to replace the drift \(r\) with \(r - \delta\). We do not introduce this payout rate for two reasons. First, we want our static model to be a straightforward extension of M74, where there are no dividend payments to equity-holders and no coupon payments to bondholders. Second, since default can be triggered only at time \(T\), it is the terminal value \(V_T\) that matters: subtracting \(\delta\) from the risk-free rate amounts to multiplying this terminal value by the constant factor \(e^{-\delta T}\).

In addition to the debt held by the market, the firm is indebted to former employees who are entitled to a payment \(L\) at time \(T\) as pension fund beneficiaries. The payment is made by the pension fund, which is assumed to be a separate entity. The pension plan is endowed with an initial asset \(A_0\), which, as it happens, can be thought of as the initial contribution from the firm, and which is invested in a risky asset \(S\) and in the bank account \(B\). In this section, we assume away interest rate risk, so the cash is risk-free at all investment horizons. Therefore, it can be seen as a perfect hedge with respect to promised liability value. Under the risk-neutral probability measure, \(S\) and \(B\) evolve as:

\[
\begin{align*}
    dS_t &= S_t[r\,dt + \sigma_S\,dz^S_t], \\
    \frac{dB_t}{B_t} &= r\,dt
\end{align*}
\]  

(1.2)

where \(z^S\) is another Wiener process whose correlation with \(z^V\) is denoted by \(\rho\). We assume away intermediate contributions from the sponsor. Therefore, the financial portfolio of the pension fund is self-financed, the only contribution from the firm to the pension plan is the initial endowment \(A_0\), and the value of the pension portfolio evolves as:

\[
\begin{align*}
    dA_t &= A_t[r\,dt + \omega_t\sigma_S\,dz^S_t] \\
\end{align*}
\]  

(1.3)

where \(\omega_t\) is the weight allocated to the risky asset. In the numerical exercises (see subsection 1.2 below), we will consider fixed-mix strategies where \(\omega_t\) is maintained constant (so that we can drop the index \(t\)). With this assumption there is a clear link.
2. A Stylized Model with Default at Terminal Date Only

between the amount of risky assets that the pension fund holds in its portfolio and the volatility of $A_T$. Indeed, we have:

$$A_T = A_0 e^{\left( r - \frac{\omega^2 x^2}{2} \right) T + \omega x z^T}$$

$V_0$ denotes the initial unlevered asset value of the firm and $x$ the initial capital available to the firm. We also let $\theta$ denote the corporate tax rate. Throughout the paper, we will assume that any contribution to the pension fund is tax deductible, which imposes the following budget constraint:

$$(1 - \theta)A_0 + V_0 = x$$

It says that the sum of the initial contribution to the pension fund, net of the tax deduction, and the capital allocated to the operating projects, equals the total amount of capital brought by the initial owners of the firm.

2.1.2 Payoffs to Stakeholders

We consider three groups of agents that hold claims on the firm and perhaps on pension fund assets: pensioners are the retired workers of the sponsor firm, equity-holders (or shareholders) own shares in the firm, and bondholders hold bonds issued by the firm. The group of pensioners, equity holders and bondholders will be referred to as the group of claimholders (or stakeholders). Following M74, we assume away any payment to either agent between dates 0 and $T$, and default can be triggered only at time $T$.

The pension fund is committed to paying the amount $L$ at date $T$ to pensioners. Hence a pension contract is a collateralized form of debt held by the pensioners of the firm, where the pension fund assets serve as a collateral. If the pension fund is insolvent (i.e., if $A_T < L$), the sponsor is called on to make a contribution $L - A_T$. If the operating assets $V_T$ are sufficient to cover the deficit, pensioners receive $L$ as promised; otherwise, default is triggered. In the opposite case, where the pension fund enjoys a surplus ($A_T > L$), this surplus is shared by pensioners and equity-holders: pensioners receive a contractual and constant fraction $1 - \gamma$ of the surplus, while the remainder, $\gamma$, net of taxes goes to equity-holders. In fact, any surplus returned to the sponsor plan is subject to a special tax regime known as "the reversion tax". New legislation was passed in 1986 regarding the tax treatment of excess pension assets: it levied a 10% excise tax on reversions from defined-benefit plans. This rate was raised to 50% in 1990, unless the sponsor gives at least 25% of the reversion to participants (in the form of contributions to some other plan), in which case the reversion tax is 20% (Ippolito 2001, 2002). The reversion (net of the excise tax) is also subject to the normal corporate tax. The amount remaining after debt payment, if strictly positive, goes to equity-holders. We let $\theta_{rev}$ be the reversion tax rate, which is equal to 50% if $\gamma = 75\%$ and 20% otherwise. In our base case we shall assume that surpluses reverted to the sponsor are not subject to taxation. To encompass both situations, we will let $\theta_{eff}$ be the effective tax rate. Hence we will have $\theta_{eff} = 0$ in the base case and $\theta_{eff} = 1 - (1 - \theta)(1 - \theta_{rev})$ in the presence of the reversion tax. If $\gamma = 0$ then the entire surplus goes to the beneficiaries of the pension plan in the form of enhanced pension benefits. On the other hand, when $\gamma = 1$ (which we shall treat as our base case in the numerical exercise), shareholders have full access to any after-tax pension fund surplus.
2. A Stylized Model with Default at Terminal Date Only

Bondholders are promised a fixed payment $D$ at time $T$. Equity-holders are responsible for redeeming debt, using the terminal unlevered value of the firm $V_T$, net of any bankruptcy costs, and a fraction of any pension plan surplus. As explained above, this fraction is equal to $(1-\theta_{\text{eff}})\gamma$. In the end, bondholders receive the promised payment if any one of the following three conditions is met: both the sponsor and the pension fund are solvent ($A_T \geq L$ and $V_T \geq D$); the pension plan is insolvent ($A_T < L$) but the sponsor can make up for the deficit ($V_T \geq D + L - A_T$); the sponsor is insolvent ($V_T < D$) but the pension fund enjoys a surplus ($A_T \geq L$) and the current tax and surplus-sharing regimes make it possible to avoid bankruptcy ($V_T + \gamma(1-\theta_{\text{eff}})(A_T - L) \geq D$).

In the other states of the world the firm is liquidated and bondholders receive a recovery payment. Since the recovery payment in the event of default may include a fraction of any pension fund surplus, it can be said that bondholders have a conditional and limited claim on pension surpluses.

As a general rule, equity-holders receive the aggregate assets of the firm and pension fund $A_T + V_T$, net of the payments to pensioners and bondholders if the firm is not in default at time $T$, and they receive zero as soon as the firm is bankrupt. They are also entitled to the tax savings at time $T$. We assume that the contributions to the pension plan as well as the interest payments on debt are tax deductible (when the firm is not in default). The amount of interest paid on debt is $D - D_p$, where $D_p = De^{-rT}$ is the present value at time 0 of the promised repayment to debtholders. Hence the tax savings on interest payments are equal to $\theta(D - D_p)$. The tax savings on contributions are $\theta(L - A_T)^+$. We now summarize the payoffs to each group of claim-holders at time $T$, in the various states of the world. For notational convenience, we let $C_T$ be the payoff of the call written on the pension fund’s assets with exercise price $L$, i.e. we set $C_T = (A_T - L)^+$. We assume that bondholders can receive the promised payment $L$.

1. $A_T \geq L$ and $V_T \geq D$: in this situation, equity-holders receive a positive payoff.

   1. $A_T \geq L$ and $V_T \geq D$: the firm pays $D$ to debt-holders, and pensioners receive $L + (1-\gamma)C_T$, that is, the promised payoff $L$ plus a fraction $1-\gamma$ of the pension fund surplus. Equity-holders receive the remaining assets of the firm plus the remaining part of the surplus net of the tax reversion, plus the tax shield. The payoff that they receive is thus $V_T - D + \gamma(1-\theta_{\text{eff}})C_T + \theta(D - D_p)$.

   2. $A_T < L$ and $V_T \geq D$: pensioners still receive $L + (1-\gamma)C_T$. Default can be avoided if $V_T + (1-\theta_{\text{eff}})C_T \geq D$, in which case bondholders receive $D$ and equity-holders $V_T - D + (1-\theta_{\text{eff}})C_T + \theta(D - D_p)$. Otherwise, bankruptcy is triggered, entailing a loss to third parties in the form of bankruptcy costs. Bondholders receive the proceeds of the liquidation, $(1-\alpha)V_T$, plus the fraction $\gamma(1-\theta_{\text{eff}})$ of the surplus, and equity-holders receive nothing.

   3. $A_T < L$ and $V_T \leq D$: in this case, the sponsor makes a final additional contribution $L - A_T$ to the pension plan so that pensioners can receive the promised payment $L$. Bondholders also receive the promised payment $D$, and equity-holders receive the remaining assets of the firm plus the tax shield, $A_T + V_T - L - D + \theta(D - D_p) + \theta(L - A_T)$.
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In this case there is no pension fund surplus to be shared by pensioners and equityholders.

II. $A_T + V_T < L + D$: in this case, equityholders receive nothing.

1. $A_T < L$ and $V_T < D$: both the firm and the pension fund default on their obligations. Pensioners receive the totality of the pension assets (collateralized nature of pension obligations) plus some fraction $q$ of the proceeds $(1−α)V_T$ of the liquidation of the firm’s assets, after bankruptcy costs. Assuming equal seniority of bondholders and pensioners, $q$ must be equal to

$$\begin{align*}
q &= \frac{L−A_T}{D+L−A_T}.
\end{align*}$$

In other words, both pensioners and debt-holders receive an amount proportional to the remaining amount due them before liquidation $(L−A_T$ for pensioners, and $D$ for bondholders). Pensioners thus receive $A_T + q(1−α)V_T$, and bondholders are paid $(1−q)(1−α)V_T$.

2. $A_T ≥ L$ and $V_T < D$: pensioners receive $L+(1−γ)C_T$. The amount available for debt payment is $V_T+γ(1−θ_{eff})C_T$, which is strictly less than $D$. Hence default is triggered: bondholders get $(1−α)V_T+γ(1−θ_{eff})C_T$, and equity-holders get nothing.

3. $A_T < L$ and $V_T ≥ D$: this case is similar to the case where $A_T < L$ and $V_T < D$: bondholders receive $(1−q)(1−α)V_T$; pensioners receive $A_T + q(1−α)V_T$.

Formally, the payoffs to each group of stakeholders can be written as:

$$\begin{align*}
L_T &= L \left[ \mathbb{1}_{\{A_T + V_T ≥ L + D\}} + \mathbb{1}_{\{A_T ≥ L, A_T + V_T < L + D\}} \right] + (1−q)(1−α)V_T \mathbb{1}_{\{A_T < L, A_T + V_T < L + D\}} + (1−γ)C_T
\end{align*}$$

$$\begin{align*}
D_T &= DT \left[ \mathbb{1}_{\{A_T + V_T ≥ L + D, A_T + V_T < L + D\}} + (1−q)(1−α)V_T \mathbb{1}_{\{A_T < L, A_T + V_T < L + D\}} + \mathbb{1}_{\{A_T ≥ L, V_T < D\}} \right]
\end{align*}$$

$$\begin{align*}
E_T &= \{V_T − D\}^+, \quad L_T = 0,
\end{align*}$$

$$\begin{align*}
D_T &= DT \left[ \mathbb{1}_{\{V_T ≥ D\}} + (1−α)V_T \mathbb{1}_{\{V_T < D\}} \right]
\end{align*}$$

which is equivalent to equations (9.c) and (11) in M74.

2.1.3 Prices of the Claims

The fair values at time 0 of the pension fund liabilities and of corporate bonds are:

$$\begin{align*}
L_0 &= \mathbb{E}^Q\left[e^{-rT}L_T\right]
\end{align*}$$

$$\begin{align*}
D_0 &= \mathbb{E}^Q\left[e^{-rT}D_T\right]
\end{align*}$$

They can be interpreted as the cost of issuing these claims for the corporation. In fact, there is one important difference between the debt held by pensioners and that held by bondholders, which is that the latter is tradable while the former is not. This implies that the value of receiving the pension claims for pensioners is, ex ante, lower than the cost of issuing such claims for the corporation. We discuss this question in the next subsection. Similarly, the price at time 0 of equity is given by:

$$\begin{align*}
E_0 &= \mathbb{E}^Q[e^{-rT}E_T]
\end{align*}$$

The actual payment $L_T$ to pensioners can be decomposed as $L_T^1 + L_T^2$, where $L_T^1$ is the...
2. A Stylized Model with Default at Terminal Date Only

sum of the first two terms on the right-hand side of (2.5) and \( L_{T}^{2} \) is the access to pension fund surpluses. With \( L_{0}^{1} \) being the present value of \( L_{T}^{1} \), the credit spread \( s_{r} \) is the rate, in excess of the risk-free rate, at which the face value should be discounted to give the fair value:

\[
L_{0}^{1} = e^{-(r+s_{r})T} L
\]

\( L_{0}^{1} \) is used rather than \( L_{0} \) in this definition, to obtain a non-negative spread. In other words, we attempt to measure the impact of default risk on the valuation of promised liability payments, while excluding possible access to pension fund surpluses. We take this definition better to compare the fair value of liabilities and the "regulatory" liability value; this regulatory value is the present value of the promised liability, \( L \), discounted at rate \( r + s_{\text{reg}} \), where \( s_{\text{reg}} \) is a spread given by the regulator:\(^{13}\)

\[
L_{0}^{1} = e^{-(r+s_{\text{reg}})T} L
\]

Dividing the current assets of the pension fund by the regulatory liability value, one obtains the regulatory funding ratio:

\[
F_{t}^{\text{reg}} = \frac{A_{t}}{L_{t}^{\text{reg}}}
\]

As for pensioners, we can define a credit spread for bondholders by:

\[
D_{0} = e^{-(r+s_{D})T} D
\]

We now define the aggregate value of the firm and the pension fund. To do this, we first introduce the present values of bankruptcy costs, tax savings, and tax reversion:

\[
BC_{0} \equiv \mathbb{E}^{Q}{e^{-rT}BC_{T}}
\]

\[
TS_{0} = \theta A_{0} + \mathbb{E}^{Q}{e^{-rT}TS_{T}}
\]

\[
RT_{0} = \mathbb{E}^{Q}{e^{-rT}RT_{T}}
\]

where:

\[
BC_{T} = \alpha V_{T} \left[ I{(A_{T}+V_{T}+D_{T}+\theta A_{T}+\omega_{T}e^{-\alpha T}D_{T})<D} + I{(A_{T}+V_{T}+D_{T})}\right] + e^{-(r+s_{D})T}D
\]

\[
TS_{T} = \theta(D - B_{0}) + \mu(A_{T}+V_{T})I(A_{T}+V_{T}+D_{T}+\theta A_{T}+\omega_{T}e^{-\alpha T}D_{T}) \geq 0
\]

\[
RT_{T} = \gamma B_{0} e^{rT}
\]

It is straightforward to verify from the payoffs of subsection 1.1.2 that we have, in any state of the world:

\[
L_{T} + E_{T} + D_{T} = V_{T} + A_{T} + TS_{T} - RT_{T} - BC_{T}
\]

(1.12)

This simply states that the aggregate assets of the firm and the pension fund, net of taxes and liquidation costs, are shared by all stakeholders at time \( T \). Taking the present value of (1.12) under the risk-adjusted probability measure \( \mathbb{Q} \), we obtain that

\[
L_{0} + E_{0} + D_{0} = V_{0} + A_{0} + TS_{0} - \theta A_{0} - BC_{0}
\]

whence:

\[
v_{0} = L_{0} + E_{0} + D_{0} = V_{0} + (1 - \theta)A_{0} + TS_{0} - RT_{0} - BC_{0}
\]

(2.13)

We refer to this quantity as the total value of the firm and pension fund; it is denoted \( v_{0} \). If the firm is not subject to taxation (\( \theta = 0 \)), no taxes are levied on surpluses (\( \theta_{rev} = 0 \)), and there are no liquidation costs (\( \alpha = 0 \)), the total value is just \( x \), and is thus independent of any choice of face value of debt, \( D \), of initial pension funding, \( A_{0}/(Le^{-rT}) \), and of portfolio allocation decision \( (\omega_{t})_{t} \), although the components \( L_{0}, E_{0}, \) and \( D_{0} \) may still depend on these parameters.

2.1.4 A Comment on Valuation of Liability Claims

We have argued that pension liabilities can be regarded as defaultable bonds held by employees. It should be recognized, however, that the private valuation of these

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13 - In the US, regulation prescribes the use of the spread between Treasury and AA-bonds.
claims for pensioners is, in general, lower than the cost for the firm of issuing these claims, and this for at least two related reasons. On the one hand, the pension claims are not tradable, and the valuation of these claims should therefore incorporate an illiquidity premium. On the other hand, employees of the firm are severely under-diversified with respect to sponsor risk, to which their human capital already shows a strong exposure. The non-tradable nature of the claim prevents employees from hedging away this risk. Similar arguments have been made for stock and stock option compensation packages, for the valuation of which preference-based pricing rules (e.g., a certainty equivalent principle) have been proposed (see Detemple and Sundaresan (1999), for example). It remains to be seen why a company puts workers at risk when it could sell this exposure at a lower cost on the bond market. One possible explanation is related to the fact that pensions are not publicly traded contracts and therefore decreases in their value might not be as obviously recognizable by pensioners as decreases in bond values from deterioration in credit ratings. One of the contributions of this paper, as it happens, is to provide pensioners and other stakeholders with a means of pricing pension claims consistently.

In the absence of information regarding preferences, endowments, trading and consumption strategies for employees, we may recognize only that $L_0$, the cost of issuing these claims for the corporation, provides an upper bound for the private value of the pension claims. One should note, however, that the difference between the value and the cost of the pension claims disappears in a complete market situation, in which the value of the claim can be obtained by arbitrage arguments only, or where the payoff generated by the claim can be replicated by pensioners through suitably designed dynamic trading strategies involving the available assets. In fact, our model is cast in a complete market setting, so any exposure to pension liability claims can be hedged away, if needed, through a suitably designed dynamic replication strategies involving two risky assets, the stock index, as well as the sponsor company stock, in addition to the risk-free asset.

One additional source of complexity here is that the value of the stock of the sponsor company as well as the pension fund asset value are endogenous and depend on the decisions made by stakeholders (contribution and leverage decisions made by the sponsor, and allocation decisions made by the pension fund). In fact, if these decisions are constant, or readjusted in a predictable manner, the pension payoffs remain attainable/replicable by trading in existing securities. In what follows, we present an extremely simplified example to explain how the replication strategy can be accommodated to account for the presence of predictable changes in the allocation policy. More general situations can be handled similarly. Let us assume, for instance, that the entire initial capital $x$ is invested in the pension fund, and that there is no corporate debt; that is, we take $V_0 = 0$ and $D = 0$. Assume, moreover, that $\gamma = 1$. Hence the payoff to pensioners is: $L_T = \min(L, A_T) = L - (L - A_T)^+$. In this extremely simplified setting, pensioners hold a short position in a put written on pension fund assets, with a strike price given by the promised liability payment. If $A$ follows a geometric Brownian motion, then it is well known that the put can be dynamically replicated using a delta-neutral
strategy. Hence $L_T$ can be replicated as well. More generally, it is sufficient to assume that $A$ has a deterministic volatility over the lifetime of the option. To see this, let us assume that the weight allocated to the risky asset $S$, $\omega_t$, is a deterministic function of time, and that the remainder is invested in the cash (as in equation (2.3)). Then it can easily be shown that the price of the put is:

$$p(t, A_t) = L e^{-r(T-t)} \mathcal{N}(-d_{2,t}) - A_t \mathcal{N}(-d_{1,t})$$

where:

$$d_{1,t} = \frac{1}{\sqrt{\int_t^T \sigma_S^2 \omega_s^2 ds}} \left[ \ln \frac{A_t}{L} + r(T-t) + \frac{1}{2} \int_t^T \sigma_S^2 \omega_s^2 ds \right]$$

$$d_{2,t} = d_{1,t} - \sqrt{\int_t^T \sigma_S^2 \omega_s^2 ds}$$

The delta of the put is $-\mathcal{N}(-d_{1,t})$, so buying $\mathcal{N}(-d_{1,t})$ shares of $A$ starting from the initial wealth $Le^{-rT} - p(0, A_0)$ yields the payoff $L_T$. Examples of situations where $\omega_t$ is deterministic include the case where it is constant (fixed-mix strategies) and the case where it is piecewise constant over time, with the jump instants being deterministic:

$$\omega_t = \sum_{i=0}^n \omega^I \mathbb{1}_{\{t_i \leq t < t_{i+1}\}}$$

where $0 = t_0 < t_1 < ... < t_n = T$ is a subdivision of the interval $[0, T]$ and the $\omega^I, i = 1, ..., n$, are constant. In the more general case where $V$ and $D$ are not zero and $\omega_t$ is a deterministic function of time, the pair $(S, V)$ follows a Markov process under $Q$, so there is a function $L$ such that $L_t = L(t, S_t, V_t)$. In this case, the payoff $L_T$ can be replicated by a dynamic trading strategy in the stock index $S$ and the stock issued by the firm $E$, if it is publicly traded (to hedge against unexpected changes in the firm asset value process $V$).

### 2.2 Numerical Results

In this section, we present some numerical results on the pricing of the claims written on the pension fund and the firm assets. These prices have been obtained by simulating the joint distribution of the final asset value and final payoff, a joint log-normal distribution given the assumption of a constant weight $\omega$ in (2.3). We assume in the base case that $\gamma = 1$, i.e., the whole pension surplus goes (after tax) to the shareholders. The initial capital $x$ is set to 100, so all prices can be directly interpreted as percentages of this quantity. The other base-case parameters are displayed in table 1.

#### 2.2.1 Leverage Decisions

We first analyze the impact of leverage decisions on total value of the firm and fair liability value, for different values of the promised payment $L$. $L$ is taken as a measure of the size of commitments to pensioners. The leverage ratio is defined as the market value of debt over the sum of debt and equity values. Since we focus on leverage choices, we set in this subsection the initial endowment of the pension fund equal to the regulatory value of liability, so the pension fund is fully funded at the starting date, at least from a regulatory standpoint. The initial contribution net of tax deduction cannot exceed the initial capital available to the firm before the pension fund is in place. This requirement implies that the set of possible promised payments is bounded from above. Precisely, the following must hold:

$$L < x \frac{e^{(r + \sigma_{reg})T}}{1 - \theta}$$

(2.14)
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Given our base-case parameters, the upper bound is equal to 253.6.

In the presence of frictions (taxes and bankruptcy costs), the trade-off theory of capital structure suggests that an optimal leverage decision can be achieved to maximize total firm value. Intuitively, one expects that a firm without a pension fund will optimally take on more debt than an otherwise identical firm sponsoring a pension plan, since the latter firm has already issued a form of debt by committing to a payment to retired employees. The results of figure 1 confirm that the total value-maximizing leverage ratio is indeed a decreasing function of L. In the absence of the pension fund (for $L = 0$), the optimal leverage ratio is 34.92%, a value that is of the same order of magnitude as what is found in related papers that use a dynamic capital structure model with a random default date (see for example Leland (1994, 1998) or Ju and Ou-Yang (2006), who find leverage ratios ranging from 30% to 50% depending on parameter values).

For sufficiently high values of the promised payments $L$ (e.g., $L = 100$ or $L = 150$), the optimal leverage ratio is in fact zero, suggesting that the amount of debt already issued in the form of pension claims makes it sub-optimal for the firm to issue any more debt to market participants. The collateralized nature of the pension obligations and the existence of potentially complex surplus-sharing rules imply, however, that debt held by pensioners and debt held by bondholders are not perfect substitutes one for another. Overall, the main insight that we obtain from figure 1 is that the existence of a pension plan should have an effect on capital structure decisions.

Figure 2 takes the reciprocal point of view and plots the credit spread $s_L$ implied by the fair liability value $L_0$ as a function of the leverage ratio. It shows that capital structure decisions will have an impact on the fair value of pension claims. Other things being equal, pensioners will prefer a sponsor with a small amount of outstanding debt to a heavily indebted one, since a more financially constrained firm is less likely to be able to afford to make additional contributions if and when needed. The impact is very substantial: for an initially fully funded pension plan, increasing the leverage ratio by a factor of 3 from 15% to 45% also leads to multiplying by a factor of 3 the defaultable liability spread $s_L$ (from less than 50 basis points to more than 150 basis points). In line with the intuition, we find that for a given level of funding the credit spread is higher if promised payments to pensioners are a large fraction of firm’s total commitments ($L = 150$) than if they are of the same order of magnitude as debt ($L = 50$). One important finding is that the regulatory valuation (based on an arbitrary spread, taken to be equal to 100 basis points in the base case) leads to overestimating the fair value of liabilities for highly leveraged firms, whereas it leads to underestimating the liability value for firms with little debt outstanding. For firms with large commitments to pensioners ($L = 150$) and a pension plan with initial funding of 70% or 100% only, liabilities are found to be undervalued, whatever the leverage ratio. While these effects are straightforward from a qualitative standpoint, one of the contributions of the model is to show that the magnitude of the over- or underestimate can be quantitatively substantial for reasonable parameter values.
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2.2.2 Allocation Decisions

We now turn to the impact of the allocation decision \( \omega \) at the pension fund on the value of the claims. Figures 3 to 6 display the market values of claims held by pensioners, bondholders, and equity-holders as functions of the pension fund allocation \( \omega \) to stocks, for different values of the regulatory funding ratio and of the promised payment to pensioners, \( L \). We consider five values for the initial regulatory funding ratio. Since the initial capital made available to the pension fund net of tax deduction cannot exceed \( x \), this ratio is bounded from above:

\[
\frac{A_0}{L_0^{\text{reg}}} < \frac{x}{(1 - \theta)L_0} e^{(r+s_{\text{reg}})T}
\]

In the base case, the right-hand side is equal to 5. When the ratio gets closer to this upper bound, the initial amount of cash available for investment in the firm activities, \( V_0 \), shrinks to zero. Finally, we consider three values of the correlation between the process \( V \) of unlevered asset value and the value of the risky asset \( S \): \( \rho = 0.5 \) (the base case), \( \rho = 0 \) and \( \rho = -0.5 \).

Figure 3 shows that the fair value of payment to pensioners, \( L_0 \), is in general a decreasing function of \( \omega \). As a first level of explanation, this can be explained by the fact that pensioners in the base case have no access to the pension fund surplus \( \gamma = 1 \), which corresponds to panel (a) in figure 3. In essence, if we assume away for a moment the presence of the sponsor, pensioners hold a short option written on the assets of the pension fund with payoff \( \min(L, A_T) \), so a riskier strategy unambiguously leads to a decrease in \( L_0 \).\(^{18}\) When the presence of the sponsor is accounted for, the situation is more subtle, since the payoff to pensioners is contingent on the realization of both \( A_T \) and \( A_T + V_T \) [see equation (2.5)]. In this context, a higher allocation to the risky asset leads to increasing the volatility of the pension fund assets, but it may also introduce a diversification effect at the total financial plus real asset value \( A_T + V_T \) level, at least when the stock index is negatively correlated to the unlevered value of the firm. This competition between the two effects is expected ex ante to lead to an interior optimal solution. As a matter of fact, we do obtain that, for a negative correlation, there is an optimal interior allocation, at least when the initial funding does not exceed 100%. When the pension fund is sufficiently funded, pensioners have no interest in the pension fund taking risks given the collateralized nature of their claims. On the other hand, the diversification effect disappears when the correlation is non-negative, which explains why \( L_0 \) then becomes a monotonically decreasing function of \( \omega \). Giving pensioners partial access to surpluses (i.e., setting \( \gamma \) to a value less than 1) introduces one additional dimension to the analysis, namely pensioner access to upside performance of the pension fund investments. Indeed, panel (b) in figure 3 shows that, when pensioners have access to the full surpluses of the fund \( \gamma = 0 \), \( L_0 \) generally becomes an increasing function of the allocation to the risky asset. This effect is more pronounced for a negative correlation because holding the risky asset diversifies away firm risk.\(^{19}\)

In the base case \( (\gamma = 1) \), equity-holders are entitled to the full surplus of the pension fund and are thus expected to benefit from
riskier investment strategies, especially when the pension plan is highly funded, which is confirmed by the panel (a) of figure 4.\textsuperscript{20} This increase in shareholder value as a function of increasing volatility of the pension fund assets through an increase in the allocation to stocks, as opposed to cash, comes at the cost of a related decrease in bondholder wealth (see figure 5), which is a clear case of asset substitution (see Jensen and Meckling (1976)).

A look at figures 3 and 5 shows obvious similarities between the fair value of the promised payment to bondholders and the fair value of pension payments, with a key difference again related to the collateralized nature of pension liabilities. That pension fund assets serve as a collateral for pension liabilities explains why the fair value of pension claims \(L_0\) is higher, other things equal, than the fair value of debt \(D_0\), or equivalently that credit spreads for corporate debt that are higher than credit spreads for pension liabilities. When funding falls, on the other hand, the diversification benefits start to become effective, provided that the correlation \(\rho\) is negative, so the volatility of the aggregate assets of firm and pension fund, \(A + V\), exhibits a minimum as a function of \(\omega\). This results in an interior optimal value for the allocation parameter.

Finally, we consider in figure 6 the overall impact of investment decisions on the firm and pension fund value \(L_0 + E_0 + D_0\). It appears that, in general, the total firm value \(V_0\) is maximized for zero investment in the risky asset, hence providing some support to the proponents of fully investing pension assets in safe instruments. It is only when the correlation \(\rho\) is negative that a riskier strategy may lead to an increase in the total value and that an interior optimal weight may exist. For initial funding of 70% or 100%, the optimal weight is around 50%, and it grows to 70% for funding of 150%.

2.2.3 Funding Decisions

Funding decisions are made by the initial owners of the firm at time 0, who decide how to allocate the initial capital \(x\) to the firm \((V_0)\) and the pension fund \((A_0)\). Figures 7 to 9 show the impact on claim values of changes in the initial contribution to the pension fund \(A_0\).\textsuperscript{21} On the whole, figure 7 confirms that pensioners always benefit from increases in funding to the pension plan because the plan assets are used as a collateral for the pension claims. This increase in the value of liabilities takes place at a decreasing rate: when the pension plan is already very well funded, marginal increases in asset value generate only marginal improvements in pensioners’ welfare. The impact on equity-holders is also straightforward: increases in funding lead to increases in benefits from the tax advantage and thus to higher shareholder value. This is confirmed in panel (a) of figure 8, which represents the base case where shareholders have full access to the pension fund surplus, at least for low face values of debt to pensioners. In this context, the tax shield effect dominates, and investing more in the pension fund is not detrimental to the sponsor, since it can enjoy negative contributions in the end. On the other hand, for large pension claims (e.g., \(L = 100\)), an increase in funding starting from low funding levels can lower the value of equity: when it is highly unlikely that pension assets will ever be in excess of pension liabilities, one additional dollar invested in the pension fund is certain to be lost for shareholders,

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who would rather have it invested in the firm. More formally, the right derivative of the shareholder value as a function of $A_0$ for $A_0 = 0$ is independent of $\gamma$; starting from a very small asset value, the likelihood of generating a surplus beneficial to shareholders is very low. Panel (b) of figure 8 shows that shareholder value is a decreasing function of the allocation to the pension fund when $\gamma = 0$, a situation where equity-holders have no access to the upside of pension asset values. Figure 9 focuses on the impact of funding decisions on bondholders' welfare. When surpluses go to pensioners alone, substantial increases in funding ratios lead to lower bond values because higher funding to the pension fund might make it harder for the firm to redeem its debt. For reasonably low funding levels, bondholders benefit from higher funding because it is in their interest that the pension fund be able to fulfill its obligations without needing additional contributions, but beyond a certain funding, the value of corporate bonds decreases with the funding level. When equity-holders have access to surpluses, on the other hand, increases in the funding ratio do not hurt bondholders because part of any pension surpluses can be transferred to them if the sponsor is not wealthy enough to pay back the debt. On the whole, figure 10 suggests that higher funding leads unambiguously to higher total firm value. When equity-holders have full access to surpluses, this comes as no surprise, because making a high initial contribution makes it possible to derive greater benefits from the tax regime, and exceedingly large surpluses can be returned to the firm so that it does not lead to increases in the likelihood of bankruptcy. When pensioners have access to surpluses, large contributions still lead to high tax shelter benefits, but they also lead to increases in bankruptcy probability. However, panel (b) of figure 10 shows that the total value is still an increasing function of the funding level, which shows that the present value of the tax savings increases faster than bankruptcy costs. In figure 11, finally, we analyze bondholder value as a function of the pension fund's initial funding ratio. As one would have expected, we find that a firm with a pension fund in surplus has a better rating quality than an otherwise comparable firm with a pension fund in deficit, and the impact of pension funding decisions on credit ratings is found to be quite substantial.22

2.2.4 Impact of Surplus-Sharing Rules

In our base-case model, equity-holders have access to full surpluses; i.e., the parameter $\gamma$ is set to 1. Figures 12 to 15 show the impact of this parameter on the prices. $\gamma$ is the fraction of any pension fund surplus that goes to equity-holders. Obviously, this parameter has a strong impact on the present values of the actual payments to pensioners and equity-holders. This can be directly seen from the payoffs (2.5) and (2.7): $L_0$ is a linearly decreasing function of $\gamma$, while $E_0$ is quasi-linearly increasing in $\gamma$. A higher $\gamma$ value is beneficial to equity-holders because they are entitled to a higher fraction of pension fund surpluses, as a result of which they are more likely to receive a positive payment at date $T$. Indeed, a higher $\gamma$ makes it possible for the firm to receive negative contributions from its pension fund at terminal date, and these contributions make default on the bond payment less likely. Of course, the impact of $\gamma$ is greater when the pension fund is highly funded, because a poorly funded pension plan is unlikely ever

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22 - That pension funding has a positive influence of credit ratings has been empirically verified by Martin and Henderson (1983) or Carroll and Niehaus (1998).
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to enjoy any surplus. Since bondholders have an indirect access to surpluses when \( \gamma \) is positive, they also benefit from a higher \( \gamma \), which can be verified on figure 14. However, this upside potential is capped because they can never receive a payment higher than the face value of debt. Hence, the impact of \( \gamma \) on the value of corporate bonds is less pronounced than on the value of equity. Finally, figure 15 measures the impact of the surplus-sharing rule on the total value of the firm. The overall effect of increasing \( \gamma \) is shown to be slightly positive. Again, the intuition is straightforward: increasing \( \gamma \) allows for larger negative contributions, thanks to which default can be avoided in some states of the world, which decreases bankruptcy costs without adversely impacting the tax shield. However, the effect on the total value is not very strong because the increase in \( E_0 \) induced by a higher fraction \( \gamma \) almost entirely offsets the related decrease in \( L_0 \).

2.3 Introducing a Pension Benefit Guarantee Corporation

The costs of corporate default have so far been assumed to be borne by the pensioners, who may not get their promised payment if the pension fund is insolvent and the sponsor is unable to cover the deficit. We now introduce a pension benefit guarantee corporation (PBGC), which is intended to provide pension benefits for pensioners of companies that are in distress. In the US, the PBGC provides protection, but the protection is limited to a statutory maximum amount ($47,659 in 2006 for employees retiring at age sixty-five). One could price this so-called pension put and obtain its rational value as a function of the pension funding status, the firm’s capital structure, and the pension fund allocation policy (see Marcus (1987), Hsieh, Chen, and Ferris (1994) and Boyce and Ippolito (2002)). If the PBGC charges sponsors a premium proportional to the actual credit riskiness of liabilities, the costs are transferred to shareholders. In practice, pension insurance is not fairly priced (the premium actually paid is independent of both capital structure and pension fund allocation), and in reality mostly underpriced for the most vulnerable companies, so motives for asset substitution (extracting value from the PBGC, that is, from other, healthier, companies) remain intact (see Sharpe (1976) or more recently Bodie (1996) for a discussion of the pension put, and also Bodie et al. (1985) for empirical evidence that the PBGC creates an incentive for distressed companies to underfund their pension plan and invest in risky assets).

2.3.1 Payoffs to Claim-holders

In the US, the actual provisions of the PBGC as defined by the Employee Retirement Income Security Act (ERISA) are as follows. The sponsoring firms with defined-benefit plans must enroll in the pension benefit insurance program of the PBGC which insures pension benefits up to a fraction of the promised payment \( L \). The fixed dollar amount of insurance premium charged by the PBGC is currently $2.60 per employee per year. Under ERISA, the PBGC can preempt assets of the sponsor, to the limit of 30% of their value. This preemptive right is senior to all unsecured liabilities of the company except wages.

We now provide a stylized model that will define the payoffs to each group of claimholders in the presence of the PBGC. Where \( A_T \geq L \) or \( A_T + V_T \geq L + D \),

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23 - One could, in principle, consider a model with two firms with different pension funding status, capital structure, and pension fund allocation policy, but which pay the same premium according to current practice, and measure the size of the wealth transfer from one firm to the other. We leave this for further research.
there is no need for the PBGC to intervene. Indeed, the assets of the firm are sufficient to cover a deficit of the pension fund. Hence the payoffs to each group are identical to the payoffs in the absence of the PBGC (see item I. in section 2.1.2). The situation is different if \( A_T + V_T < L + D \) and \( A_T < L \). Here the firm does not have enough assets to make up for the deficit of the plan, so liquidation takes place. Then the PBGC can withdraw a fraction \( p \) of the assets net of bankruptcy costs, where: \( p = \min (0.3, q) \),

where \( q = \frac{L - A_T}{D + L - A_T} \) \hspace{1cm} (2.15)

Putting together pension fund assets and a fraction \( p \) of the firm assets, the amount available to compensate pensioners is \( A_T + p(1-\alpha) V_T \). Since this quantity is less than \( L \), the PBGC must intervene to close the gap, up to a fraction of the face value of liabilities. Finally, pensioners and bondholders receive, respectively:

\[ v_0^\text{PBG} = x - P_0^\text{eq} + P_0 + T S_0 - B C_0 - R T_0 \] \hspace{1cm} (2.16)

Comparing this expression and the total value in the absence of a guarantee fund (see (2.13)), we obtain that:

\[ v_0^\text{PBG} = v_0 + P_0 - P_0^\text{eq} \]

If the PBGC sells the put at fair value, i.e., if \( P_0^\text{eq} = P_0 \), the value of the firm with the guarantee is equal to the value without the guarantee. But the share of total firm value that goes to each of the three groups of claim-holders is still impacted by the presence of the PBGC. In practice, the premium that the PBGC charges the sponsor is not equal to the fair value of the put, and is taken equal to a fraction \( \eta \) of the regulatory value \( \tilde{l}_0^\text{reg} \). The total value of the firm will be increased if the pension put is underpriced (\( \eta \)) and decreased if it is overpriced (\( \eta \)). However, in our model, maximizing the total value in the presence of the PBGC with respect to corporate and pension fund policies and maximizing the difference between the fair and the regulatory values of the pension put are not equivalent objectives. Indeed, the first term \( v_0 \) is in general also impacted.
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by these policies. Were the frictions equal to zero, \( v_0 \) would be equal to \( V_0 \), and the equivalence would be recovered. This is an important difference between our model and Sharpe (1976), who derives optimal policies by maximizing the excess of the fair value of the insurance over the premium paid by the sponsor.

2.3.2 Numerical Results

In this subsection, we study the impact of the PBGC on the values of the various claims. The base-case parameters are those of table 2. We take \( \lambda = 0.85 \) as in Bicksler and Chen (1985) and \( \eta = 0.1 \). This choice of parameter values implies that the PBGC charges the sponsor an amount equal to 10% of the regulatory value \( L_0^{\text{req}} \), and that its contribution is capped to 85% of promised pension payments. In the base-case situation, where equity-holders have full access to pension surpluses, figure 16 shows that the presence of the PBGC makes pensioners become almost indifferent to the asset allocation policy of the pension fund. It is only for very low funding ratios and very aggressive strategies that they can experience a loss, so the credit spreads \( s_i \), which are virtually zero in all other cases, then become slightly positive. Equity-holders and bondholders are also impacted by the introduction of the pension guarantee fund, even though the impact is less substantial, as can be seen by comparing figures 4 and 17 on the one hand, and figures 5 and 18 on the other.

The strategy maximizing the pension put, a strategy discussed in (Sharpe 1976) or Bicksler and Chen (1985), suggests that it is optimal for shareholders to maximize the benefits obtained from the mispricing of the pension put, which is at its peak when the allocation to the risky asset is maximum, as confirmed in figure 19). For a negative value of the correlation parameter \( \rho \), there is in fact an interior value of the allocation that minimizes the value of the pension put. This minimum exists because the volatility of the aggregate assets of firm and pension fund is minimal for some non-trivial allocation to the risky asset. Figure 19 also suggests that the difference between the fair value of the pension put \( P_0 \) and the regulatory value for the pension put \( P_0^{\text{req}} \) can be positive or negative depending on parameter values, and can be quite substantial for reasonable parameter values. Looking at the overall effect on total firm and pension value (see figure 20), we see that only extreme allocations are optimal, with an optimal allocation that can be \( \omega = 0 \) or \( \omega = 1 \), depending on parameter values.

The next series of figures (from 21 to 24) focus on the impact of funding decisions in the presence of the PBGC. As was to be expected, figure 21 shows that the fair value of the payment to pensioners is very weakly impacted by these decisions, as long as surpluses go to equityholders. It is only for vanishing levels of funding that credit spreads become strictly positive, although they are still much smaller than in figure 7. Comparing figures 8 and 22 reveals that the impact of funding decisions on equity-holders is not substantially different in the presence and the absence of the guarantee corporation. However, bondholders are more impacted when the promised payment to pensioners is large (more than 100% of the initial capital), as can be seen from figures 9 and 23. These are the situations where the premium charged by the PBGC to the sponsor firm is particularly high, which reduces the initial unlevered value of the firm (see the budget constraint (2.16)). Moreover, with a large debt to pensioners, the pension
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fund is unlikely to enjoy surpluses that could eventually help the sponsor repay its debt. This results in a non-monotonic pattern, with the price of corporate bonds being a decreasing function of the funding level for low or medium funding ratios. The same effects are at work for the total value (see figure 24). For large promised payments to pensioners, this aggregate quantity exhibits a U-shape. Similar results have been obtained by Bicksler and Chen (1985), in the context of a model with pension termination costs and a progressive tax structure on corporate income. For our base-case parameter values, we obtain that funding to the maximum, as opposed to the minimum, is the optimal solution from a perspective of maximizing firm value.
3. Extending the Model to an Intertemporal Setting
3. Extending the Model to an Intertemporal Setting

We now consider a number of extensions to the base-case model, which was purely static. We first relax the assumption of fixed-mix strategies to analyze dynamic risk-controlled strategies. Subsequently, we extend the model to account for the presence of a random default date of the sponsor, as well as minimum funding requirements.

3.1 Dynamic Risk-Controlled Strategies

We have considered so far fixed-mix asset allocation strategies. It has been shown, however, that some particular forms of risk-controlled state-dependent strategies would be utility-maximizing in the presence of funding ratio constraints (Martellini and Milhau 2008). In a related effort, Bodie (1990a) discussed the benefit of contingent immunization for an overfunded pension plan. These papers, however, have addressed the problem in a simplified setting where the pension fund situation was analyzed somewhat in isolation from the sponsor. In this section, we test for the impact of introducing dynamic risk-controlled strategies on stakeholder welfare. For simplicity, we focus here on basic constant proportion portfolio insurance (CPPI) strategies, which of course have no particular reason to be optimal from a perspective of maximizing total firm value (see section 4 for a discussion of more general strategies that can prove to generate more substantial welfare improvement).

3.1.1 CPPI Strategies

The previous literature has considered two main families of risk-controlled strategies: the constant proportion portfolio insurance (CPPI) strategies and the option-based portfolio insurance (OBPI) strategies. CPPI strategies were introduced by Black and Jones (1987) and Black and Perold (1992), and they have the advantage over OBPI strategies of leading to a very simple and intuitive expression for the weight allocated to the risky asset: this weight is a multiple, denoted by $m$, of the outstanding risk budget, defined as the difference between the current wealth level and some minimum wealth level known as the floor. The commitments of the pension fund suggest a simple form for the minimum level: since pensioners in this simplified model are promised a fixed payment $L$ at time $T$, we define the floor as the present value of the promised payment, $Le^{-r(T-t)}$. More generally, one can consider a floor equal to a fraction $k$ of the previous present value. The risk budget will then be a decreasing function of $k$, and the weight allocated to the risky asset will be:

$$\omega_t = m \frac{A_t - kL_t}{A_t}$$

(3.17)

where $m$ denotes the multiplier, and $k$ can be interpreted as some minimum funding ratio requirement. As shown in appendix A, such strategies can be rationalized in a reduced-form expected utility framework, which also makes it possible to relate the multiplier to the risk aversion of the investor and to the Sharpe ratio of the risky asset. Further details, including an extension to the case of multiple asset classes, can be found in Basak (2002) and in Martellini and Milhau (2008). Having defined a strategy by (3.17), one can write the state-dependent terminal wealth achieved by an investor following this strategy, as shown in the proposition below.
Proposition 3.1 Assume that the interest rate $r$ and volatility $\sigma_S$ are both constant, and that the pension fund follows the strategy (3.17). Terminal wealth is then given by:

$$A_T = kL + (A_0 - kLe^{-rT}) \exp \left[ (1 - m) \left( r + \frac{m\sigma_S^2}{2} \right) T \right] \left( \frac{S_T}{S_0} \right)^m$$

**Proof.** See appendix B.

In particular, if the initial condition

$$A_0 \geq kLe^{-rT} \quad (3.18)$$

holds, then the terminal funding ratio, $A_T/L$, is greater than $k$ almost surely, at least in the context of a continuous-time implementation. If $k$ is taken equal to or greater than 1, the pension fund will be able to deliver the promised payment $L$ in any state of the world, without requiring any unexpected contribution from the sponsor. If $k$ is less than 1, implementing the CPPI strategy does not ensure that the funding ratio will end up greater than 1, but at least the deficit $L - A_T$ will be bounded from above by $(1 - k)L$. This assigns a limit to the size of the contribution that the sponsor could have to make in the event of a deficit.

In practice, the possible choice for $k$ will be constrained by the initial contribution $A_0$. Condition (3.18) indeed implies that $k$ cannot exceed $A_0/(Le^{-rT})$, or, put in terms of regulatory quantities, $F^\text{eq}_0 e^{\text{reg} T}$. If $A_0$ is greater than the discounted promised payment to pensioners, $k$ can be set at a level greater than 1. Of course, starting from such a funding level, the pension fund might also deliver the payment $L$ with full certainty by investing in cash alone, but it would be unable to take advantage of the performance of the risky asset. 26 The above risk-controlled strategy implemented with a minimum funding level $k$ ensures that there will never be a deficit exceeding $1-k$, but also opens access to some upside performance. If the initial endowment $A_0$ is less than the discounted promised payment, setting $k$ to its maximum value cannot ensure that the final funding ratio will be greater than 1, but the size of the worst deficit will be limited to $(1 - k)L$, while the upper bound would be $L$ in the absence of risk management. The choice of the multiplier allows one to define the riskiness of the strategy: the higher the multiplier, the higher the volatility of the payoff. The case where $m$ is zero corresponds to an investment in the risk-free asset only, whereas the case where $m = 1$ corresponds to a buy-and-hold strategy: starting from the initial weight $A_0$, one buys $kLe^{-rT}$ shares of the risk-free asset, and one invests the remaining wealth $A_0 - kLe^{-rT}$ in the stock index. Beyond these fixed-mix and buy-and-hold strategies recovered as special cases, any value for $m$ strictly greater than 1 defines a truly dynamic risk-controlled strategy.

When the above risk-controlled strategy is implemented with a minimum terminal funding ratio $k$ at least as large as 1 and an initial funding compatible with (3.18), the pension fund ends up more than fully funded in all states of the world. This leads to many simplifications in the payoffs to stakeholders (see equations (2.5), (2.6) and (2.7)). In particular, with our base-case parameter values (see table 1), equity-holders have access to surpluses ($\gamma = 1$) and the sponsor pays no taxes on these surpluses ($\theta_{\text{ctf}} = 0$). Therefore, pensioners receive the promised payoff:

$$L_T = L \quad (3.19)$$

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26 - See also appendix B for a comparison of the risk-controlled strategy and the strategy investing in cash alone in a mean-variance setting.
3. Extending the Model to an Intertemporal Setting

On the other hand, equity-holders hold a long position in a call written on $A+V$, plus a long position in a digital option paying the tax savings on interest payments if the firm does not default on its debt:

$$E_T = [A_T + V_T - L - D + \theta(D - D_p)] 1_{\{A_T + V_T \geq L + D\}}$$

Finally, the payoff to bondholders becomes:

$$D_T = D 1_{\{A_T + V_T \geq L + D\}} + [(1 - \alpha)V_T + A_T - L] 1_{\{A_T + V_T < L + D\}}$$

3.1.2 Numerical Exercise

In this subsection, we conduct an empirical investigation of the impact of introducing a CPPI strategy on the prices of the claims. We take the minimum funding ratio $k$ to be equal to 1. Given condition (3.18), this implies that the initial regulatory funding ratio must satisfy $F_{0\text{reg}}^\text{req} \geq e^{S_{0\text{reg}}} T$. With our base-case values, this lower bound is equal to 1.11, so the pension fund must be more than fully funded in the regulatory sense. For each possible initial weight $\omega_0$, the multiplier $m$ is chosen according to the following rule, which ensures that the initial allocation to the risky asset is between 0 and 1:

$$m = \frac{\omega_0}{1 - k F_{0\text{reg}}^\text{req} e^{S_{0\text{reg}}} T}$$

where $F_{0\text{reg}}^\text{req}$ is the initial regulatory funding ratio. We also limit the multiplier to values less than 5, which is considered a maximum value by most practitioners. Thus, the set of possible multipliers depends on the initial funding level. For example, the highest multiplier is 3.8 for $F_{0\text{reg}}^\text{req} = 200\%$; is approximately equal to 2.24 for $F_{0\text{reg}}^\text{req} = 150\%$; and is 5 for $F_{0\text{reg}}^\text{req} = 130\%$.

Unsurprisingly, figure 25 shows that pensioners are completely insensitive to the choice of the multiplier, which is consistent with (3.19). Indeed, the pension fund delivers $L$ in all states of the world, and since they have no access to surpluses, they have no appetite for performance. On the other hand, figure 26 shows that for high funding ratios of 150% and 200%, equity-holders are willing to increase the riskiness of the investment strategy by increasing the multiplier $m$. This behavior jibes with the insights obtained with the fixed-mix strategies (see figure 4), and is mainly explained by the fact that shareholders are assumed to have (full) access to surpluses in our base case, and that it is thus in their interest to engage in riskier investment strategies. The main insight that we get from this analysis is that implementing risk-controlled strategies aiming at insuring a minimum funding ratio allows shareholders some (limited) access to the upside performance of risky assets, while ensuring that pensioners will not be substantially hurt by the induced increase in risk. Indeed, increasing the value of the multiplier $m$ does not impact pensioners, while it leads to an increase in equity value, due to the access to performance. In the fixed-mix case, increasing the weight allocated to the risky asset also benefited equity-holders, but was in general detrimental to pensioners, except when there was a sufficiently negative correlation between the firm value process and the stock index value process. The effect of diversifying firm risk and market risk, which exists only for a negative correlation $\rho$, is still present, and may account for the local minimum of $E_0$ that we observe around $m = 1$ for a...
funding ratio of 130%. As for the fixed-mix strategies, a riskier strategy leads to a decrease in bondholder welfare, but this side effect is less pronounced. This can be seen from figure 5, where we have reported the relative change in $D_0$ defined as:

$$\frac{D_0(CPPI) - D_0(\text{fixed mix})}{D_0(\text{fixed mix})}$$

When the correlation parameter $\rho$ is non-negative, bondholders' claims are worth more than they are when the pension fund follows a fixed-mix strategy. From the aggregate perspective (see figure 28), an increase in the multiplier has ambiguous effects: taking more risk has a positive impact on the total value for a funding ratio of 150%, an impact which turns out to be negative for a funding ratio of 200%. Here again, the intuition is that the implicit opportunity cost of downside risk protection may not be worth paying when insurance against downside risk is highly likely not to be needed in the first place. However, if the multiple is set to its highest value (subject to the constraint of an initial allocation to the risky asset less than 100%), which maximizes the access to surpluses for equity-holders, the total value of the firm is higher than if the pension fund were following the strategy of investing only in the risky asset.

3.2 A Fully Dynamic Capital Structure Model

As noted in the introduction, the literature has, for the most part, considered the valuation of pension liabilities in a simplified setting where default is either assumed away or happens only at terminal date, as in Merton (1974). Although they provide useful insight, these papers cannot address features such as minimum funding ratio constraints. In this section we extend the previous model to account for the presence of intermediate payments, random default date of the sponsor, and minimum funding ratio constraints.

3.2.1 Introducing Intermediate Payments and a Random Default Date

So far we have considered only zero-coupon debt contracts, promising fixed payments at the terminal date $T$ to pensioners and bondholders. In practice, pensioners receive periodic pension payments and bondholders receive coupon payments. It is straightforward to introduce these payments in the model. We assume that bondholders are promised a coupon payment $c$ to time $T$, plus the face value of debt $D$ at this date, whereas pensioners are promised a payment $l$ to time $T$, plus a final lump-sum payment $L$. The coupon and pension payments are assumed to be continuous for notational convenience, but accounting for discrete payments would involve no additional difficulties.

The present value of the promised payments to pensioners is denoted by $L_t^p$, and is given by:

$$L_t^p = \int_t^T t e^{-r(s-t)} ds + L e^{-r(T-t)}$$

It is to be distinguished from the fair value, denoted by $L_t$, of actual pension payments after incorporation of default risk as well as surplus sharing, the estimation of which is one of the focus of the paper. An ad hoc way to account for the presence of default risk in the valuation of the liability stream is to use an exogenously specified spread, as recommended by the regulator. This leads to
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the regulatory value of pension liabilities:

\[ L_{t}^{eq} = l \int_{t}^{T} e^{-(r+s_{eq})(s-t)} \, ds + L e^{-(r+s_{eq})(T-t)} \]

Similarly, the present value of the promised payments to bondholders is:

\[ D_{t}^{p} = c \int_{t}^{T} e^{-r(s-t)} \, ds + D e^{-r(T-t)} \]  

(3.20)

Because of the presence of a stream of promised payments to pensioners, we also introduce a continuous stream of contractual contributions \( \kappa \), paid by the sponsor to the pension fund. For simplicity, \( \kappa \) is taken constant, but the actual contribution may differ from this quantity, given the presence of regulatory funding requirements, as will be explained below (see subsection 3.2.2). The coupon payments to bondholders and the contributions to the pension fund will impact the firm’s unlevered value. This can be taken into account by introducing a payout rate \( \delta \) in the drift of \( V \). In the absence of funding ratio constraints, we would thus have:

\[ dV_t = V_t[(r - \delta) \, dt + \sigma \sqrt{V} \, dz_t] \]  

(3.21)

where \( \delta = c + \kappa + d \), and \( d \) is the dividend rate paid to equity-holders. The dynamics (3.21), however, will be slightly modified below (see subsection 3.2.2) to account for the existence of minimum funding constraints and contribution holidays. We now assume that default is triggered when the unlevered value of the firm falls below a fraction \( \beta \in [0, 1] \) of \( D_{p}^{p} \):29

\[ \tau = \inf \{ t \geq 0 ; V_t \leq \beta D_{t}^{p} \} \]  

(3.22)

This notion of default is consistent with that of Briys and de Varenne (1997), who define the default date as the first time unlevered value falls below a fraction of the present value of face value of debt.30 Given that the pension fund assets are not held on the sponsor company balance sheet, it should be noted that they have no direct impact on the definition of default. Conversely, the presence of default risk will obviously have an impact on the liability value. Indeed, early default by the sponsor will force a liquidation of the pension fund before terminal date \( T \).

3.2.2 Introducing Minimum Funding Ratio Constraints

In the US, the Employment Retirement Income Security Act (ERISA), enacted in 1974 and subsequently complemented by several rounds of related legislation, establishes a system of mandatory pension contributions when the deficit of the pension plan exceeds given threshold values. The presence of minimum funding ratio constraints at intermediate dates, which exist in most developed countries (see Pugh (2003)), was not accounted for in the previous model, which allowed only for the presence of contributions at the initial and terminal dates. We now take into account such required contributions, which increase the probability of default on the non-pension debt and depress capital investment by the sponsor (Rauh 2006), as a potentially important ingredient of the

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29 - Taking \( \beta = 0 \), which we take as our base-case value in the numerical analysis, corresponds to the absence of protective covenants. 30 - Other papers have considered default barriers close to ours, but assume that \( \beta = 1 \) (see Leland (1994) and Ju and Ou-Yang (2006)).
model. Our objective is not to derive the optimal dynamic contribution strategy, a question that is arguably difficult to address given the complexity of the underlying model, but to analyze a strategy consisting of contributing only if, as required by regulation, minimum funding ratios are breached.

We now describe a series of contributions inspired by the regulatory practice of checking the funding status of the pension fund at predefined dates \(0 < t_1 < \ldots < t_N\). We assume that there is a regulatory minimum floor whose value at time \(t\) is denoted by \(F_t\). If the assets of the pension fund just before time \(t\), \(A_{t^-}\), are found to be lower than the minimum funding \(F_{t^-}\), the sponsor is called on to contribute. This contribution is equal to a fraction \(a_1\) of the current deficit: \(a_1(F_{t^-} - A_{t^-})^{+}\). Taking \(a_1 = 1\) implies that the sponsor should make up for the entire deficit immediately, while taking \(a_1 < 1\) allows for a partial recovery.\(^{31}\)

On the other hand, the contractual contribution can be decreased when the pension fund enjoys sufficiently large surpluses. By large surpluses, we mean that \(A_{t^-} > G_{t^-}\), where \(G\) is some stochastic adapted process such that \(G > F\). In this case, the sponsor benefits from what is known as a contribution holiday: the normal contribution stream, denoted by \(\kappa dt\), is decreased by \(a_2(At^- - G_{t^-})\). This covenant implies that pensioners have partial access to large pension fund surpluses. In what follows, we define \(F\), \(G\), and \(H\) as proportions of the regulatory liability value \(L^\text{reg}\):

\[
F_t = kL_t^\text{reg}, \quad G_t = k'L_t^\text{reg}, \quad H_t = k''L_t^\text{reg}
\]

with \(k < k' < k''\). In particular, these processes are continuous, so that:

\[
F_{t^-} = F_{t^-}, \quad G_{t^-} = G_{t^-}, \quad H_{t^-} = H_{t^-}.
\]

The previous covenants imply that the pension fund asset value \(A\) evolves as:

\[
dA_t = rA_t dt + \Delta \sigma \sigma S d\tau_t - \kappa dt
\]

\[
+ \sum_{i=1}^{N} \sigma_i(kL_t^\text{reg} - A_{t^-})^{+} dM_t^i
\]

\[
- \sum_{i=1}^{N} \sigma_i(A_{t^-} - kL_t^\text{reg}) 1_{(A_{t^-} > kL_t^\text{reg})} dM_t^i
\]

\[
+ \sum_{i=1}^{N} (k - a_2(A_{t^-} - k'L_t^\text{reg})^{+}) \tau_t 1_{(A_{t^-} < k'L_t^\text{reg})} dt
\]

with \(dM_t^i\) is the Dirac measure at time \(t\):

\[
dM_t^i = \begin{cases} 1 & \text{if } s = t \\ 0 & \text{otherwise} \end{cases}
\]

For further use, let \(\kappa t dt\) be the overall continuous contribution rate:
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\[ \kappa_t = \sum_{i=1}^{N} \left( \kappa - \alpha_2 \left( \kappa_t - k'L_{t_i}^{\text{req}} \right)^+ \right)^t \mathbb{I}_{\{t_i < t \leq t_{i+1}\}} \]

d\kappa_t = \kappa_t dt + \sum_{i=1}^{N} \alpha_1(kL_{t_i}^{\text{req}} - A_{t_i})^+ dM_{t_i}^h

which may differ from \( \kappa dt \) if the sponsor enjoys a contribution holiday or if some exceptional contribution takes place.

The dynamics of \( A \) in our model will, in general, differ from the dynamics of financial wealth in papers that consider asset allocation with labor income (see Bodie, Merton, and Samuelson (1992) and Munk and Sørensen (2005)) because of the obligation for the sponsor to contribute \( a_1 \neq 0 \) and the existence of contribution holidays \( a_2 \neq 0 \). Moreover, the asset \( A_t \) may fall below the regulatory floor \( F_t \) between two consecutive checking dates \( t_i \) and \( t_{i+1} \).

On the whole, the dynamics of the unlevered value of the firm are given by a modified version of (3.21) in which we have accounted for exceptional contributions:

\[ dV_t = V_t \left[ (r - \delta) dt + \sigma_V dz_t^V \right] - \sum_{i=1}^{N} \alpha_1(kL_{t_i}^{\text{req}} - A_{t_i})^+ dM_{t_i}^h \]

with \( \delta V_t = c + \kappa_t + d_t \). Like \( A, V \) is a right-continuous process and can exhibit discontinuities only at points \( t_i \).

3.2.3 Payoffs to Stakeholders

We now provide a detailed analysis of the actual payoff accruing to each stakeholder. The last payments to each group of stakeholders (pensioners, equity-holders and bondholders) take place at time 
\[ \tau_1 = \tau \wedge T, \] and those are contingent on the values of \( A \) and \( V \) just before \( \tau \). The pension fund is committed to paying the amount \( L \) at date \( T \) to pensioners. Hence a pension contract is a collateralized form of debt held by the employees and pensioners of the firm, where the pension fund assets serve as a collateral. If the pension fund is insolvent at terminal date (i.e., if \( A_T < L \)) or at default date (i.e., if \( A_T < L_T^D \)), the assets of the firm are used to compensate pensioners. Of course, these assets may not be sufficiently high to cover the deficit of the pension plan, in which case the firm is technically in default (default triggered by the pensioners).

As in section 2, we assume the presence of a rule by which pensioners and equity-holders share the surplus if the firm has not defaulted prior to time \( T \): if the pension fund enjoys a surplus at time \( T \), pensioners receive a contractual fraction \( 1 - \gamma \) of this surplus, whereas the remainder, \( \gamma \), goes to equity-holders. In the first case the entire surplus goes to the beneficiaries of the pension plan in the form of enhanced pension benefits. On the other hand, when \( \gamma = 1 \) (which we treat as our base case), shareholders have full access to any (after tax) pension fund surplus (see below for the tax treatment of pension reversion). More generally, equity-holders are responsible for debt payment, using the terminal unlevered value of the firm \( V_T \) plus a fraction of any pension plan surplus. However, in practice, surpluses that reverted to the sponsor are subject to a special tax regime, as described in subsection 2.1.2. As in section 2, we let \( \theta \) be the normal corporate tax rate, \( \theta_{\text{rev}} \) the reversion tax rate, and \( \theta_{\text{eff}} \)
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the effective tax rate on surpluses. In our base case we will assume that reverted surpluses are not subject to taxation, so \( \theta_{\text{eff}} \) will be set to zero.

The amount remaining after debt payment, if strictly positive, goes to equity-holders. Default is triggered by bondholders when the actual payment made to them is lower than the promised payment. Since the recovery payment in the event of default may include a fraction of the pension fund surplus (assuming \( \gamma > 0 \)), it can be said that bondholders have a conditional and limited claim on pension surpluses.

We thus have the following covenants:

1. If \( \tau > T \): then we have that \( \tau_1 = T \). We assume that, in this case, any pension fund surplus, is shared by equity-holders and pensioners, before debt repayment. The shareholders receive the fraction \( \gamma \) of the surplus, while the the pensioners receive the fraction \( 1 - \gamma \). If the sum of firm assets and of the fraction \( \gamma \) of the surplus is insufficient to repay corporate debt, equity-holders must cede their access to the surplus to bondholders. That is, in the event of bankruptcy, overfunded pension plans can be terminated, with the excess assets distributed to the creditors (see Ippolito (2001), footnote 10). The payoffs to the various claimholders in this situation are identical to the payoffs in the base model (see subsection 2.1.2);

2. If \( \tau \leq T \): we then have that \( \tau_1 = T \). The firm is liquidated and equity-holders receive nothing. By definition of and by right-continuity of \( V \) and \( D^p \), we have that \( D^p, V_r \leq BD^p \leq D^p \). In this case, we assume that equity-holders receive nothing, whatever the value of \( \gamma \), and that any pension fund surplus is shared by bondholders and pensioners.

(a) If \( A_{\tau-} \geq L^p_{\tau} \): bondholders receive \( D_{\tau-} = \min(D^p_{\tau_1}, A_{\tau-} - L^p_{\tau} + (1 - \alpha)V_{\tau-}) \) and pensioners receive the remaining assets of the pension fund \( L_{\tau_1} = A_{\tau-} - \min(D^p_{\tau_1} - (1 - \alpha)V_{\tau-}, A_{\tau-} - L^p_{\tau}) \);

(b) If \( A_{\tau-} < L^p_{\tau} \): pensioners receive \( A_{\tau-} + q(1 - \alpha)V_{\tau-} \) and bondholders receive \( (1 - q)(1 - \alpha)V_{\tau-} \), where \( q = \frac{L^p_{\tau} - A_{\tau-}}{D^p_{\tau} + L^p_{\tau} - A_{\tau-}} \).

3.2.4 Fair Values of the Claims and Total Value of the Firm

Having written the payoffs received by each group of claimholders, we can write their present values and define the total value of the firm and the pension fund. Bondholders receive the coupon payment to time \( \tau_1 \), plus a terminal payment \( D_{\tau_1} \), so the present value of the claim that they hold is:

\[
D_0 = \mathbb{E}^Q \left[ \int_0^{\tau_1} e^{-r t} c \, dt \right] + \mathbb{E}^Q \left[ e^{-r \tau_1} D_{\tau_1} \right]
\]

(3.25)

and the corresponding credit spread \( s_D \) is implicitly defined by the relation:

\[
D_0 = c \int_0^T e^{-(r+s_D)s} ds \, dt + De^{-(r+s_D)T}
\]

It is straightforward to show that \( D_0 \) is smaller than \( D^p_0 \) as defined in equation (3.20), so the previous definition ensures that \( s_D \) is indeed non-negative.

On the pension fund side, the present value
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of actual pension payments is:

\[
L_0 = \mathbb{E}^Q \left[ \int_0^{\tau_1} e^{-rt} \ell \, dt \right] + \mathbb{E}^Q \left[ \sum_{i=1}^{N} a_3 e^{-rt} (A_{t_i} - F_{ti}) \mathbbm{1}_{\{A_{t_i} > H_{t_i}\}} \mathbbm{1}_{\{t_i < \tau_1\}} \right] + \mathbb{E}^Q \left[ L_{\tau_1} e^{-r\tau_1} \right]
\]

To define a credit spread for the claim held by pensioners, we decompose the present value of this claim, as given in (3.26), into two terms: the first, \(L_0^1\) is the present value of the actual payments excluding access to pension fund surpluses, whereas the second, \(L_0^2\), is the present value of the access to these surpluses. We have that:

\[
L_0^1 = \mathbb{E}^Q \left[ \int_0^{\tau_1} e^{-rt} \ell \, dt \right] + \mathbb{E}^Q \left[ e^{-r\tau_1} \min(L_{\tau_1}, L_0^2) \right]
\]

\[
L_0^2 = \mathbb{E}^Q \left[ \sum_{i=1}^{N} a_3 e^{-rt} (A_{t_i} - F_{ti}) \mathbbm{1}_{\{A_{t_i} > H_{t_i}\}} \mathbbm{1}_{\{t_i < \tau_1\}} \right] + \mathbb{E}^Q \left[ e^{-r\tau_1} (L_{\tau_1} - L_0^2)^+ \right]
\]

The credit spread \(s_L\) is then the value of the regulatory spread that would be needed to make the regulatory value equal to the fair value:

\[
L_0^1 = L \int_0^T e^{-(r+s_L)t} \, dt + Le^{-(r+s_L)T}
\]

Again, it can be verified that \(s_L\) is a non-negative quantity. As far as they are concerned, equity-holders receive the dividend payment \((\delta V_t - \kappa_t - c) \, dt\) to time \(\tau_1\) and the payoff \(E_{\tau_1}\) at time \(\tau_1\).

Hence the fair value of equity: is:

\[
E_0 = \mathbb{E}^Q \left[ \int_0^{\tau_1} e^{rt} \delta V_t - (1 - \theta)(\kappa_t + c) \, dt \right] + \theta \mathbb{E}^Q \left[ \int_0^{\tau_1} \sum_{i=1}^{N} e^{-rt} a_1 (k(t_i)^{\text{req}} - A_{t_i})^+ \, dM^I_{t_i} \right] + \mathbb{E}^Q \left[ E_{\tau_1} e^{-r\tau_1} \right]
\]

(3.27)

The total value of the firm and the pension fund is defined as:

\[
v_0 = L_0 + D_0 + E_0 \quad (3.28)
\]

This is the aggregate fair value of the pensioners', bondholders', and equity-holders' claims to firm and pension fund assets. Intuitively, this should also be the aggregate value of operating assets and pension fund assets, augmented by the tax shield and decreased by the present value of bankruptcy costs and other tax payments. This property indeed holds here, as will be shown in the next proposition. Before formulating this result, we introduce a few notations. The (present value of ) bankruptcy costs is defined by:

\[
BC_0 = \mathbb{E}^Q \left[ e^{-r\tau_1} BC_{\tau_1} \right]
\]

where \(BC_{\tau_1}\) is the amount lost to bankruptcy procedure at time \(\tau_1\), namely:

\[
BC_{\tau_1} = \alpha V_{\tau_1} \mathbbm{1}_{\{\tau \leq \tau_1\}} + \alpha V_{\tau_1} \mathbbm{1}_{\{\tau > \tau_1\}} \left( \mathbbm{1}_{\{A_T + V_T < L + D\}} \right.
\]

\[
\left. + \mathbbm{1}_{\{A_T + V_T \geq L + D\}} \right) \quad (3.27)
\]

The tax shield is defined as the present value
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of future tax savings from contributions and coupon payments, plus the tax savings from the initial contribution:

\[ TS_0 = \theta A_0 + \theta E^Q \left[ \int_0^{\tau_1} e^{-r t} (c + \kappa_t) dt + \int_0^{\tau_1} \sum_{i=1}^N e^{-r t} a_1 (k L_i^{\text{reg}} - A_{t_i^+})^+ d M_t^b \right] + E^Q \left[ e^{-r \tau_1} TS_{\tau_1} \right] \]

where \( TS_{\tau_1} \) is the tax savings corresponding to the last contribution, if any, made at time \( \tau_1 \):

\[ TS_{\tau_1} = \theta (L - A_T)^+ 1_{\{T \geq \tau_1 \}} 1_{\{A_T + V_T \geq L + D, V_T \geq D \}} \]

Finally, the present value of the tax reversion is given by:

\[ RT_0 = E^Q \left[ e^{-r \tau_1} RT_{\tau_1} \right] \]

where:

\[ RT_{\tau_1} = \gamma \theta \text{eff} 1_{\{\tau > T \}} (A_T - L)^+ \]

Proposition 3.2 The total value of the firm satisfies:

\[ v_0 = (1 - \theta) A_0 + V_0 + TS_0 - BC_0 - RT_0 \]

Proof. See appendix C.

Because of the presence of minimum funding ratio constraints and the intermediate payments, there is no closed-form expression for the prices. In the numerical exercise of the next subsection, we have thus run Monte-Carlo simulations to estimate these prices.

3.2.5 Numerical Analysis

In the numerical application, we choose to focus on the impact of short-term funding constraints and consider a simplified version of the model, in which there is no intermediate payment to claimholders, no contractual contribution stream \( \kappa \) and no surplus sharing at intermediate dates. We assume that the minimum funding ratio is \( k_1 = 100\% \), the funding ratio is checked every year, and the sponsor has to cover a fraction \( a_1 = 1/3 \) of the difference between the actual value of pension assets and the regulatory floor. These assumptions are those that prevail in the Netherlands (see Pugh (2003)), where the sponsor is given three years to make up for a deficit. We impose no protective covenant on debt: that is, the parameter \( \beta \) in (3.22) is set to 0. As a consequence, default can be triggered prior to time \( T \) only at a checking date at which the pension fund posts a deficit and the sponsor cannot afford to make the regulatory contribution. The other parameter values are set as in table 1. As a result, the only difference between the results we obtain here and those obtained in the base case are the result of minimum funding ratio constraints, which allows us to analyze their marginal impact.

First, figure 29 shows that in the presence of short-term constraints, pensioners are less sensitive to the choice of the investment strategy than in the absence of such constraints (see figure 3). For a negative correlation \( \rho \) between the firm value risk and stock index risk, one still obtains an interior optimal allocation for initial funding ratios of 70% and 100%, but liability value is a flatter function of the allocation to the risky asset compared to figure 3, where no minimum funding was imposed. The intuition behind this result is that short-term constraints protect pensioners from the insolvency risk of the pension fund, regardless of the funding status or the investment policy of the pension fund. Unlike risk-controlled strategies, they do not guarantee that the
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promised payment will be delivered to pensioners in all states of the world, for any riskiness of the strategy, so the weight \( \omega \) still has an impact, but this effect is largely dampened. As far as equity-holders and bondholders are concerned, \( \omega \) also still has an impact, which is qualitatively similar to what was obtained in figures 4 and 5. Finally, figure 32 shows that, in the presence of short-term constraints, a 100% allocation to the risky asset is more often optimal from the aggregate perspective than in the absence of these constraints (see figure 6).

The intuition is that the introduction of minimum funding requirements allows for more risk taking, which is beneficial to shareholders but does not make a great impact on pensioner welfare. In figures 33 to 36, we measure the impact of introducing short-term constraints for several rules for the sharing of surpluses by pensioners and equity-holders. The rule is determined by the parameter \( \gamma \), giving the fraction of surpluses that goes to equity-holders after liquidation of the pension plan at terminal date. The impact is measured in terms of the relative change, defined as:

\[
L_0(\text{with constraints}) - L_0(\text{without constraints})
\]

for pensioners and similarly for other quantities. Unsurprisingly, figure 33 shows that, for any value of \( \gamma \), pensioners benefit from the enforcement of minimum funding requirements. This benefit could be expected from the comparison of the credit spreads in figures 3 and 29, which shows that spreads are significantly reduced when short-term constraints are introduced. The highest benefits are to be expected when the initial funding ratio is low, because it is then that the pension fund is unlikely to deliver the promised payment without needing additional contributions. The situation is more ambiguous for equity-holders. On the one hand, it can be argued that early contributions make insolvency of the pension fund less likely, thus reducing bankruptcy costs while having a positive impact on the tax shield. On the other hand, early additional contributions may turn out, after the fact, to have been unnecessary in those states of world where the fund can recover a better funding situation simply because of strong stock market performance, without calling for exceptional contributions. As these intuitions suggest, figure 34 shows that equity-holders will benefit from the introduction of short-term constraints only if \( \gamma \) is sufficiently close to 1. Indeed, if they are entitled to a large fraction of the surpluses, they can expect unnecessary contributions to be returned to them at the terminal date. If \( \gamma \) is lower, however, any such contribution is lost for them. Thus, the main message from this analysis is that equity-holders are hurt by the introduction of short-term funding constraints if there is no surplus-sharing rule (or at least a contribution holiday covenant) allowing some form of negative contributions at terminal date. Again, the introduction of surplus-sharing schemes makes it possible to align the interests of stakeholders more closely. The impact of short-term constraints on bondholders depends on the value of correlation \( \rho \) of the stock market performance and the firm asset value. Interestingly, we find that for a negative \( \rho \), they benefit from such constraints. Indeed, intermediate contributions lead equity-holders to invest more in the risky asset \( S \) through
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the pension fund, which diversifies away firm risk if the firm has a negative beta with the market. This results in lower volatility for the aggregate asset $A + V$, which increases the value of corporate bonds. Such a diversification effect is not present for a positive correlation. Here, the presence of short-term constraints will, in general, hurt bondholders because they deteriorate the financial health of the sponsor, unless the pension fund can return surpluses to the firm. Finally, figure 36 shows that in most situations introducing short-term constraints will increase the total value of the firm. These results shed new light on the fierce debate between advocates of tighter pension fund regulation, not only in the US but also in Europe, and those arguing that it would result only in a severe welfare loss. The introduction of funding ratio constraints has been particularly criticized by a number of experts, who find that imposing such short-term constraints on long-term investors could be counter-productive (see Pugh (2003)). Our results suggest that the introduction of short-term funding constraints could in fact be a welfare-improving mechanism, not only for pensioners but also for shareholders, as long as some form of explicit surplus-sharing rule is put in place.
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4. Conclusions and Extensions

Correctly assessing the value of a pension plan requires a comprehensive model for the joint quantitative analysis of capital structure choices, pension fund allocation decisions, and their impact on rational pricing of liability streams. This paper is an attempt to analyze the valuation of pension liabilities regarded as defaultable claims issued by the sponsor to workers and pensioners in the context of an intertemporal capital structure model with contingent contributions. Our model has important policy implications in that it provides a first step towards a much-needed methodological framework for the design of firm-specific regulatory constraints and accounting valuation principles. It also has a number of implications in terms of investment decisions for the pension fund, and funding decisions for the sponsor.

Our work can be extended in a number of directions. First, the model could usefully be extended to account for the fact that, in most countries, promised pension payments are indexed to price or wage inflation, in accordance with pre-specified rules. An analysis of the impact of (conditional) inflation indexation on the value of stakeholder claims would therefore be a worthwhile extension of this paper. In terms of allocation strategies, we have tested in this paper fixed-mix strategies as well as basic CPPI strategies and have found that the benefits of moving away from static allocation strategies to consider even the simplest form of dynamic risk-controlled strategies were substantial, in particular for liability-holders. It would be useful to try to test more sophisticated forms of welfare-improving strategies in a more general dynamic context, including strategies with a floor given as a function of the liability portfolio value (with a key distinction to be made between the regulatory floor and the fair value floor), strategies with a performance cap in addition to floors, which can enable a decrease in the cost of downside risk protection, as well as strategies involving corporate bonds in the liability-hedging portfolio. Moreover, one also would like to test strategies based on risk controls that encompass state variables related to the sponsor. The intuition is that one could further decrease the cost of downside protection by relaxing risk constraints when the sponsor is financially healthy and could therefore make up for eventual deficits through increased contributions, while focusing on hedging away the states of the world characterized by joint occurrences of poorly performing pension assets and ill financial health on the part of the sponsor. We leave these questions for further research.
5. Appendices
A. Optimality of Portfolio Insurance

We assume that the Sharpe ratio $\lambda_S$ of the risky asset $S$ is constant. The dynamics of $S$ under the actual probability measure $P$ is:

$$\frac{dS_t}{S_t} = [r + \sigma_S \lambda_S] dt + \sigma_S d\mathcal{Z}^S_t$$

where $d\mathcal{Z}^S_t = z^S_t - \lambda_S t$ defines a $\mathbb{P}$-Brownian motion. Let us now consider an agent (e.g., a pension fund manager) who invests the weight $\omega_t$ at time $t$ in $S$ and the remainder in cash, and maximizes expected utility from the random variable $\frac{A_T}{L} - k$, which is the excess of the funding ratio at time $T$ over the minimum funding requirement $k$. We assume that her preferences are represented by a Constant Relative Risk Aversion utility function, with relative risk aversion $\nu$. The maximization program at time $t$ can be written as:

$$\max_{\omega_t \geq t} \mathbb{E}^P \left[ \frac{1}{1 - \nu} \left( \frac{A_T}{L} - k \right)^{1-\nu} \right]$$

This problem can be solved via the duality technique of Cox and Huang (1989). First, the optimal terminal wealth is:

$$A^*_T = kL + (u M_T)^{-\frac{1}{\nu}}$$

where $u$ is some Lagrange multiplier, given by $\mathbb{E}^P [M_T A_T] = M_T A_T$ and $M_t$ is the pricing kernel at time $t$:

$$M_t = e^{- \left( r + \frac{\lambda_S^2}{2} \right) T + \lambda_S \mathcal{Z}^S_t}$$

The optimal wealth at time $t$ is thus:

$$A^*_t = kLe^{-r(T-t)} + u^{-\frac{1}{\nu}} M_t^{-\frac{1}{\nu}} \mathbb{E}_t \left[ \left( \frac{M_T}{M_t} \right)^{1-\frac{1}{\nu}} \right]$$

Applying Ito’s lemma and matching the diffusion term in $dA^*_t$ with $A^*_t \omega_t \sigma_S d\mathcal{Z}^S_t$, we obtain the optimal instantaneous allocation to the risky asset at time $t$:

$$\omega^*_t = \frac{1}{A^*_t} \left[ A^*_t - kL e^{-r(T-t)} \right] \frac{\lambda_S}{\nu \sigma_S}$$

which is of the form (3.17) with $m = \frac{\lambda_S}{\nu \sigma_S}$.

Introducing the optimal terminal wealth when no constraint is imposed (i.e., when $k = 0$), $A^*_T = (u_0 M_T)^{-\frac{1}{\nu}}$, we obtain that:

$$A^*_T = kL + \left( 1 - \frac{kL e^{-rT}}{A_0} \right) A^*_T$$

In particular, in those states of the world where $A^*_T$ is greater than $A_0 e^{rT}$, $A^*_T$ will be lower than $A^*_T$. Therefore, there is an opportunity cost associated with downside risk protection if it proves ex post to have been unnecessary.

It is also interesting to compare in a mean-variance setting the properties of the insurance strategy and those of a strategy taking no risk at all. It can easily be checked that the expected value of the payoff $A^*_T$ under the physical measure is:

$$\mathbb{E}^P [A^*_T] = kL + \left( A_0 e^{rT} - kL \right) e^{m \sigma_S \lambda_S T}$$

As soon as the Sharpe ratio $\lambda_S$ is positive and the initial risk budget satisfies (3.18), this expected value exceeds the expected terminal wealth from a strategy invested in the cash only (for which the expected payoff would be $A_0 e^{rT}$). Of course, the variance of $A^*_T$ will also exceed the zero variance that would be achieved by investing in the cash alone, but investors with non-zero risk aversion may prefer to take some risk for more average return.
5. Appendices

B Proof of Proposition 3.1
We start from the dynamics of $A$:

$$dA_t = m(A_t - kLe^{-r(T-t)}) \frac{dS_t}{S_t} + \left[A_t - m(A_t - kLe^{-r(T-t)})\right] r \, dt$$

and we then introduce $Y_t \equiv \frac{(A_t - kLe^{-r(T-t)})}{S_t^m}$.

An application of Ito’s lemma shows that:

$$dY_t \over Y_t = (1 - m) \left[r + \frac{m\sigma_S^2}{2}\right] dt$$

A straightforward integration then shows that, for any $t \leq T$:

$$A_t = kLe^{-r(T-t)} + (A_0 - kLe^{-rT}) \exp \left[(1 - m) \left[r + \frac{m\sigma_S^2}{2}\right] t\right] \left(S_t \over S_0\right)^m$$

Hence:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[\theta V_t + (\theta(k_t + c) - c_t)\right] dt \right] + \mathbb{E}^Q \left[ \int_0^T \theta \sum_{i=1}^N e^{-rt} a_i \left(k_t^{\text{req}} - A_{t-} \right)^+ dM_t^i \right]$$

C. Proof of Proposition 3.2
From (3.26), (3.25), (3.27) and (3.28), the total value of the firm satisfies:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[\theta V_t + (\theta(k_t + c) - c_t)\right] dt \right] + \mathbb{E}^Q \left[ \int_0^T \theta \sum_{i=1}^N e^{-rt} a_i \left(k_t^{\text{req}} - A_{t-} \right)^+ dM_t^i \right]$$

Using the dynamics of $A$ and $V$, as given in (3.23) and (3.24), we can rewrite the integrals in the first two terms, and arrive at:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[\theta V_t + (\theta(k_t + c) - c_t)\right] dt \right]$$

The expectations of the integrals in $dz^S$, $dz^r$ and $dz^V$ are zero, and the covenants listed in subsection 3.2.3 imply that $L_{t_1} + D_{t_1} + E_{t_1} = A_{t_1} + V_{t_1} + TS_{t_1} - BC_{t_1} - RT_{t_1}$. 

Hence:

$$v_0 = -\mathbb{E}^Q \left[ \int_0^T d \left[ e^{-rt} (A_t + V_t) \right] \right] + \mathbb{E}^Q \left[ \int_0^T \theta(k_t + c) e^{-rt} dt \right] + \mathbb{E}^Q \left[ \int_0^T \theta \sum_{i=1}^N e^{-rt} a_i \left(k_t^{\text{req}} - A_{t-} \right)^+ dM_t^i \right] + \mathbb{E}^Q \left[ e^{-rt} \left( A_{t_1} + V_{t_1} + TS_{t_1} - BC_{t_1} - RT_{t_1} \right) \right]$$

which implies that $v_0 = (1 - \theta)A_0 + V_0 + TS_0 - BC_0 - RT_0$. 

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5. Appendices

**Figure 1:** Leverage decisions with fixed-mix strategies in the absence of the PBGC – Total value of the firm as a function of the leverage ratio.

This figure plots the leverage ratio against the total value for different values of the promised payment to pensioners, $L$. The curves are parametrized by the face value of debt, $D$. The initial endowment of the pension fund is set to the regulatory liability value, i.e., we set $A_0 = L^{\text{PBGC}}$, and the initial unlevered value of the firm is set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Other parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the leverage ratio in the base case, where $L = 50$.

**Figure 2:** Leverage decisions with fixed-mix strategies in the absence of the PBGC – Fair liability value as a function of the leverage ratio.

This figure plots the leverage ratio against the credit spread $s_L$ for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and two values of the promised payment to pensioners (50 and 150). The curves are parametrized by the face value of debt, $D$. The initial endowment of the pension fund is set to the regulatory liability value, i.e., we set $A_0 = L^{\text{PBGC}}$, and the initial unlevered value of the firm is set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. The case where $L = 150$ and the initial funding ratio is 200% has not been represented because it is not compatible with the condition of a positive $V_0$. Other parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the leverage ratio in the base case, where $L = 50$, and the horizontal dashed-dot line represents the regulatory liability spread, which is taken to be 100 basis points.
Figure 3: Allocation decisions with fixed-mix strategies in the absence of the PBGC – Impact on pensioners.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\omega$. 
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Figure 4: Allocation decisions with fixed-mix strategies in the absence of the PBGC – Impact on equity holders.
(a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\omega$.

Figure 5: Allocation decisions with fixed-mix strategies in the absence of the PBGC – Impact on bondholders.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\omega$. 
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Figure 6: Allocation decisions with fixed-mix strategies in the absence of the PBGC – Impact on aggregate firm and pension fund.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\omega$.

Figure 7: Funding decisions with fixed-mix strategies in the absence of the PBGC – Impact on pensioners.

(a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures plot the regulatory funding ratio of the pension fund against the fair liability value for different values of the promised payment to pensioners, $L$ (25, 50, 100 and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.
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Figure 8: Funding decisions with fixed-mix strategies in the absence of the PBGC – Impact on equity holders.
(a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures plot the regulatory funding ratio of the pension fund against the market capitalization of the firm for different values of the promised payment to pensioners, $L$ (25, 50, 100 and 150). The curves are parametrized by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.
Figure 9: Funding decisions with fixed-mix strategies in the absence of the PBGC – Impact on bondholders. (a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures plot the regulatory funding ratio of the pension fund against the market value of debt for different values of the promised payment to pensioners, $\ell$ ($25, 50, 100$ and $150$). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.
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Figure 10: Funding decisions with fixed-mix strategies in the absence of the PBGC – Impact on aggregate firm and pension fund. (a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures plot the regulatory funding ratio of the pension fund against the total value of the firm for different values of the promised payment to pensioners, $L$ (25, 50, 100 and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.

Figure 11: Funding decisions with fixed-mix strategies in the absence of the PBGC – Debt value as a function of the regulatory funding ratio.

This figure plots the regulatory funding ratio against the market value of debt and the credit spread $s_D$. The curves are parametrised by the initial endowment to the pension fund, $A_0$. Other parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the regulatory funding ratio in the base case.
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Figure 12: Surplus sharing rule – Impact on pensioners.

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\gamma$.

Figure 13: Surplus sharing rule – Impact on equity holders.

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\gamma$.

Figure 14: Surplus sharing rule – Impact on bondholders.

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\gamma$. 
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Figure 15: Surplus sharing rule – Impact on aggregate firm and pension fund.

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1). The vertical dashed line identifies the base case for $\gamma$.

Figure 16: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on pensioners.

(a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case for $\omega$. 
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Figure 17: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on equity holders.
(a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0 - \delta t$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case for $\omega$.

Figure 18: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on bondholders.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \delta)A_0 - \delta t$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case for $\omega$. 
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Figure 19: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on the pension put.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta) A_0 - P^{\text{US}}$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case for $\omega$ and the dash-dot line represents the regulatory premium $P^{\text{US}}$.

Figure 20: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on aggregate firm and pension fund.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta) A_0 - P^{\text{US}}$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case for $\omega$. 
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Figure 21: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on pensioners.
(a) Equity holders have access to surpluses ($\gamma = 1$).
(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures plot the regulatory funding ratio of the pension fund against the fair liability value for different values of the promised payment to pensioners, $L$ (25, 50, 100 and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0 - P^{TM}$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.
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Figure 22: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on equity holders.

(a) Equity holders have access to surpluses ($\gamma = 1$).

(b) Pensioners have access to surpluses ($\gamma = 0$).

These figures plot the regulatory funding ratio of the pension fund against the market capitalization of the firm for different values of the promised payment to pensioners, $L$ (25, 50, 100 and 150). The curves are parametrized by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0 - P_L^{100}$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.

Figure 23: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on bondholders.

These figures plot the regulatory funding ratio of the pension fund against the market value of debt for different values of the promised payment to pensioners, $L$ (25, 50, 100 and 150). The curves are parametrized by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0 - P_L^{100}$ where $x$ is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.
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Figure 24: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on aggregate firm and pension fund.

These figures plot the regulatory funding ratio of the pension fund against the total value of the firm for different values of the promised payment to pensioners, \( L \) (25, 50, 100 and 150). The curves are parametrised by \( A_0 \), the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to \( V_0 = x - (1 - \theta)A_0 - \rho P \) where \( x \) is a constant normalized to 100. Unless otherwise indicated, other parameters are fixed at their base case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.

Figure 25: Allocation decisions with risk-controlled strategies in the absence of the PBGC– Impact on pensioners.

These figures perform comparative static analysis with respect to the multiplier, \( m \), for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta)A_0 \) where \( x \) is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), and the minimum terminal funding ratio \( k \) is set to 100%. The vertical dashed line identifies the base case for \( \omega \).

Figure 26: Allocation decisions with risk-controlled strategies in the absence of the PBGC– Impact on equity holders.

These figures perform comparative static analysis with respect to the multiplier, \( m \), for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta)A_0 \) where \( x \) is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), and the minimum terminal funding ratio \( k \) is set to 100%. The vertical dashed line identifies the base case for \( \omega \).
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Figure 27: Allocation decisions with risk-controlled strategies in the absence of the PBGC – Impact on bondholders. (a) Prices.

(b) Comparison with fixed-mix strategies.

These figures perform comparative static analysis with respect to the multiplier, m, for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta)A_0 \) where \( x \) is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), and the minimum terminal funding ratio \( k \) is set to 100%. The vertical dashed line identifies the base case for \( \omega \).

Figure 28: Allocation decisions with risk-controlled strategies in the absence of the PBGC – Impact on aggregate firm and pension fund. (a) Prices.

(b) Comparison with fixed-mix strategies.

These figures perform comparative static analysis with respect to the multiplier of the CPPI strategy, \( m \), for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta)A_0 \) where \( x \) is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), and the minimum terminal funding ratio \( k \) is set to 100%. The vertical dashed line identifies the base case for \( \omega \).
5. Appendices

Figure 29: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on pensioners.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$.

Figure 30: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on equity holders.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$. 
5. Appendices

Figure 31: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on bondholders.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$.

Figure 32: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on aggregate firm and pension fund.

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$.

Figure 33: Surplus sharing rule in the presence of short-term constraints – Impact on pensioners.

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity holders, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\gamma$. 
These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity holders, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150% and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta) A_0$ where $x$ is a constant normalized to 100. Unless otherwise indicated, parameters are fixed at their base case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\gamma$. 

5. Appendices
5. Appendices

Table 1: Base case parameters in the absence of the PBGC.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm</td>
<td></td>
</tr>
<tr>
<td>$e_V$</td>
<td>0.30</td>
</tr>
<tr>
<td>$V_0$</td>
<td>100</td>
</tr>
<tr>
<td>$D$</td>
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</tr>
<tr>
<td>Pension Fund</td>
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<tr>
<td>$e_S$</td>
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</tr>
<tr>
<td>$\omega$</td>
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<tr>
<td>$A_0$</td>
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<tr>
<td>$l$</td>
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<tr>
<td>Default</td>
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<tr>
<td>Regulatory Environment</td>
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<tr>
<td>$\theta$</td>
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<tr>
<td>Horizon</td>
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</table>

This table lists the parameter values in the base model. The unlevered asset value of the firm, $V$, starts at $V_0$ and has volatility $e_V$. The risky security in which the pension fund invests has volatility $e_S$, and its returns have instantaneous correlation $\rho$ with changes in $V$. The short-term interest rate is equal to $r$, and the promised payments to pensioners and bondholders at time $T$ are respectively $L$ and $D$. The pension fund is fully funded in the regulatory sense, that is the initial endowment $A_0$ is equal to the regulatory liability value. $V_0$ is then computed as $V_0 = 100 - (1 - \theta)A_0$, where $\theta$ denotes the corporate tax rate. $\theta_{eff}$ denotes the effective tax reversion rate, and $\alpha$ is the proportional rate of bankruptcy costs. At date $T$, the fraction $\gamma$ of pension fund’s surpluses goes to equity holders, and pensioners receive the remaining part $1 - \gamma$.

Table 2: Base case parameters in the presence of the PBGC.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Firm</td>
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<tr>
<td>$D$</td>
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</tr>
<tr>
<td>Pension Fund</td>
<td></td>
</tr>
<tr>
<td>$e_S$</td>
<td>0.30</td>
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<tr>
<td>$\omega$</td>
<td>0.50</td>
</tr>
<tr>
<td>$A_0$</td>
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</tr>
<tr>
<td>$l$</td>
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<td>Interest rate</td>
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<td>Correlation</td>
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<td>Default</td>
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<td>Regulatory Environment</td>
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</tr>
<tr>
<td>$\theta$</td>
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<tr>
<td>$\theta_{eff}$</td>
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</tr>
<tr>
<td>$z_{reg}$ (bp)</td>
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</tr>
<tr>
<td>$s_{reg}$</td>
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<tr>
<td>$\lambda$</td>
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<tr>
<td>PBGC</td>
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</tr>
<tr>
<td>$l$</td>
<td>0.85</td>
</tr>
<tr>
<td>$s_{PBGC}$</td>
<td>0.10</td>
</tr>
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</table>

This table lists the parameter values in the base model. The unlevered asset value of the firm, $V$, starts at $V_0$ and has volatility $e_V$. The risky security in which the pension fund invests has volatility $e_S$, and its returns have instantaneous correlation $\rho$ with changes in $V$. The short-term interest rate is equal to $r$, and the promised payments to pensioners and bondholders at time $T$ are respectively $L$ and $D$. The pension fund is fully funded in the regulatory sense, i.e. the initial contribution $A_0$ equals the regulatory liability value. $V_0$ is then computed as $V_0 = 100 - (1 - \theta)A_0 - \lambda P^{reg}_{PBGC}$, where $\lambda$ denotes the corporate tax rate and $P^{reg}_{PBGC} = \eta P^{reg}_{PBGC}$ is the premium charged by the PBGC. $\theta_{eff}$ denotes the effective tax reversion rate, and $\alpha$ is the proportional rate of bankruptcy costs. At date $T$, the fraction $\gamma$ of pension fund’s surpluses goes to equity holders, and pensioners receive the remaining part $1 - \gamma$. $s_{reg}$ denotes the regulatory spread used to valuate pension payments. $\lambda$ is the fraction of the promised payment to pensioners at which the PBGC caps its contribution.
6. References
6. References

6. References


6. References


6. References


6. References
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management

EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context. This strategic choice is applied to all of the centre’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

An Applied Research Approach

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre’s advisory board. This board is made up of internationally recognised researchers, the centre’s business partners and representatives of major international institutional investors. The management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:

- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the three EDHEC-Risk locations in London, Nice, and Singapore.

In addition, EDHEC-Risk has developed close partnerships with a small number of sponsors within the framework of research chairs. These research chairs involve a three-year commitment by EDHEC-Risk and the sponsor to research themes on which the parties to the chair have agreed.

About EDHEC-Risk Institute

The following research chairs have been endowed to date:

- **Regulation and Institutional Investment, in partnership with AXA Investment Managers (AXA IM)**
- **Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners**
- **Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS**
- **Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)**
- **Private Asset-Liability Management, in partnership with ORTEC Finance**
- **Dynamic Allocation Models and New Forms of Target-Date Funds, in partnership with UFG**
- **Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage**
- **Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank**
- **Core-Satellite and ETF Investment, in partnership with Amundi ETF**
- **The Case for Inflation-Linked Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie**
- **Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan**

Each year, EDHEC-Risk organises a major international conference for institutional investors and investment management professionals with a view to presenting the results of its research: EDHEC Risk Institutional Days.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 40,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 500,000 readers.

### EDHEC-Risk Institute: Key Figures, 2008–2009

<table>
<thead>
<tr>
<th>Category</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of permanent staff</td>
<td>47</td>
</tr>
<tr>
<td>Number of research associates</td>
<td>17</td>
</tr>
<tr>
<td>Number of affiliate professors</td>
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</tr>
<tr>
<td>Overall budget</td>
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<tr>
<td>External financing</td>
<td>€5,900,000</td>
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<tr>
<td>Number of conference delegates</td>
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<tr>
<td>Number of participants at EDHEC-Risk Executive Education seminars</td>
<td>371</td>
</tr>
</tbody>
</table>

**Research for Business**

The centre’s activities have also given rise to executive education and research service offshoots.

EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.
About EDHEC-Risk Institute

The EDHEC-Risk Institute PhD in Finance
The EDHEC-Risk Institute PhD in Finance at EDHEC Business School is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC Business School, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the European financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

The EDHEC-Risk Institute MSc in Risk and Investment Management
The EDHEC-Risk Institute Executive MSc in Risk and Investment Management is designed for professionals in the investment management industry who wish to progress, or maintain leadership in their field, and for other finance practitioners who are contemplating lateral moves. It appeals to senior executives, investment and risk managers or advisors, and analysts. This postgraduate programme is designed to be completed in seventeen months of part-time study and is formatted to be compatible with professional schedules.

The programme has two tracks: an executive track for practitioners with significant investment management experience and an apprenticeship track for selected high-potential graduate students who have recently joined the industry. The programme is offered in Asia—from Singapore—and in Europe—from London and Nice.

FTSE EDHEC-Risk Efficient Indices
FTSE Group, the award winning global index provider, and EDHEC-Risk Institute launched the FTSE EDHEC Risk Efficient Indices at the beginning of 2010. The index series aims to capture equity market returns with an improved risk/reward efficiency compared to cap-weighted indices. The weighting of the portfolio of constituents achieves the highest possible return-to-risk efficiency by maximising the Sharpe ratio (the reward of an investment per unit of risk).

EDHEC-Risk Alternative Indexes
The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a “true and fair” view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC Risk was the first to launch composite hedge fund strategy indexes as early as 2003.

The thirteen EDHEC-Risk Alternative Indexes are published monthly on www.edhec-risk.com and are freely available to managers and investors.
About BNP Paribas Investment Partners
About BNP Paribas Investment Partners

BNP Paribas Investment Partners is the dedicated autonomous asset management business line of the BNP Paribas Group.

BNP Paribas Investment Partners offers a full range of investment management services to institutional and retail clients around the world. Central to the way we work is the concept of partnership—both in terms of how we behave as a family of companies and our relationships with our clients. Around 1,000 investment professionals work across our network of some 60 investment centres, each of which is a specialist in a particular asset class or type of product. With total assets under management of EUR 539 billion (USD 736 billion) as of 30 September 2010, BNP Paribas Investment Partners is the third-largest asset manager in Europe and the ninth-largest in the world.¹

BNP Paribas Investment Partners combines the financial strength, distribution network and focus on compliance of its parent company with the reactivity, specialisation and entrepreneurial spirit of investment boutiques.

BNP Paribas Investment Partners provides a broad range of expertise and local solutions from its various Partners across the world:

- Fundamental management of the major assets classes (BNP Paribas Asset Management)
- Quantitative global equity (Alfred Berg)
- Global and emerging fixed income (FFTW)
- Funds of hedge funds (Fauchier Partners)
- Currency management (Overlay Asset Management)
- Private equity (BNP Paribas Private Equity)
- Infrastructure management (Antin Infrastructure Partners)
- Multi-management (FundQuest)
- Environmental markets (Impax Asset Management, BNP Paribas Clean Energy Partners)
- Non-listed real estate (BNP Paribas Asset Management)
- Wealth management (CamGestion, BNP Paribas Discretionary Portfolio Management)
- Employee retirement and saving schemes (BNP Paribas Epargne & Retraite Entreprises)
- Australia (Arnhem Investment Management)
- Brazil (BNP Paribas Asset Management Brasil)
- Chile (BancoEstado Administradora General de Fondos)
- China (HFT Investment Management Co. Ltd.)
- India (BNP Paribas Mutual Fund)
- Indonesia (PT. BNP Paribas Investments Partners Indonesia)
- Morocco (BMC Gestion)
- Russia (TKB BNP Paribas Investments Partners)
- Saudi Arabia (SAIB BNP Paribas Asset Management)
- South Korea (Shinhan BNP Paribas AMC)
- Turkey (TEB Asset Management, Fortis Investments Turkey)²

On April 1, 2010, the operations of Fortis Investments were merged with those of BNP Paribas Investment Partners. Fortis Investments’ investment experts and international locations are a natural and complementary fit with BNP Paribas Investments Partners, whose flexible partnership model has proven successful in integrating new expertise in the past. Together, our combined company provides clients with an even broader range of investment solutions and even better client service than before.

BNP Paribas Investment Partners has offices in the world’s major financial centres, including Hong Kong, London, New York, Paris and Tokyo.
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- Martellini, L., and V. Milhau. From deterministic to stochastic life-cycle investing: implications for the design of improved forms of target date funds (September).
- Sender, S. EDHEC Survey of the Asset and Liability Management Practices of European Pension Funds (June).
- Amenc, N., F. Goltz, and A. Grigoriu. Risk control through dynamic core-satellite portfolios of ETFs: Applications to absolute return funds and tactical asset allocation (January).
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• Amenc, N. Quelques réflexions sur la régulation de la gestion d’actifs (June).
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• Liou, A. The undesirable effects of banning short sales (April).

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• Amenc, N., and V. Le Sourd. Les performances de l’investissement socialement responsable en France (December).
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• Amenc, N., B. Maffei, and H. Till. Les causes structurelles du troisième choc pétrolier (November).

• Amenc, N., B. Maffei, and H. Till. Oil prices: The true role of speculation (November).
• Till, H. The oil markets: Let the data speak for itself (October).
• Sender, S. QIS4: Significant improvements, but the main risk for life insurance is not taken into account in the standard formula (February). With the EDHEC Financial Analysis and Accounting Research Centre.

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• Amenc, N. Three early lessons from the subprime lending crisis (August).
• Amenc, N., W. Géhin, L. Martellini, and J.-C. Meyfredi. The myths and limits of passive hedge fund replication (June).
• Sender, S., and P. Foulquier. QIS3: Meaningful progress towards the implementation of Solvency II, but ground remains to be covered (June). With the EDHEC Financial Analysis and Accounting Research Centre.
• Hedge fund indices for the purpose of UCITS: Answers to the CESR issues paper (January).
• Géhin, W. The Challenge of hedge fund measurement: A toolbox rather than a Pandora’s box (January).
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or by e-mail to: carolyn.essid@edhec-risk.com