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Bringing Research Insights to Institutional Investment Professionals

Multi-Dimensional Risk and Performance Analysis for Equity Portfolios

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Outline

- Portfolio Risk & Performance Analysis with Factors versus Attributes
- From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)
- Targeting Market Neutrality with Fundamental versus Historical Betas
Portfolio Risk & Performance Analysis with Factors versus Attributes

From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

Targeting Market Neutrality with Fundamental versus Historical Betas
Risk and Performance Analysis for Equity Portfolios

- Factor models, supported by equilibrium (CAPM, ICAPM) or arbitrage arguments (APT), are key cornerstones of asset pricing theory.

- They also have long been used in practice to perform risk and performance analysis of equity portfolios.

- On the performance side, factor models allow us to distinguish between abnormal return (alpha) and normal return (beta).

- On the risk side, factor models allow us to distinguish between specific risk and systematic risk, from either an absolute or relative risk perspectives.
Factors versus Attributes

- In addition to analyzing the impact of common factors, equity portfolio managers are also interested in analyzing the role of stock-specific attributes in explaining differences in risk & performance across assets and portfolios.

- Typical approach: turn attributes into factors.

- For continuous attributes (e.g., fundamental characteristics)
  - Academic approach: Use L/S portfolios of sorted groups of stocks as factors – Fama-French factors are not factors, they are attributes turned into factors!
  - Industry (BARRA) approach: Use normalized attribute value as proxy for factor loading.

- Quid for discrete attributes (such as sector or country)?
Discrete Attributes: Example of Sector or Country Effects

- Returns to L/O sector portfolios are highly correlated with market returns which raises multicolinearity problems.

- Possible cures:
  - Orthogonalize sector or country returns against market before running the multivariate regression, so as to isolate their specific effects:
    \[ r_{kt} = a_k + b_k MKT_t + \nu_{kt} \quad \text{for sector } k \text{ at date } t \]
    \[ r_t = \alpha + \beta MKT_t + \gamma_1 (a_1 + \nu_{1t}) + \cdots + \gamma_{10} (a_{10} + \nu_{10,t}) + \eta_t \quad \text{for a fund} \]
  - This approach is taken by Menchero and Poduri (2008) to add "custom factors" to a set of risk factors when factors are highly correlated.

- In any case, treating attributes as factors severely, and somewhat artificially, increases the number of factors to consider, especially in the case of discrete attributes.
Attributes Should Remain Attributes: From Historical to Fundamental Beta

- Instead of artificially adding new factors, one would like to maintain a parsimonious factor model and treat attributes as auxiliary variables to estimate the betas with respect to risk factors.

- Our aim is to decompose market factor exposure (beta) and risk adjusted performance (alpha) in a forward looking way as a function of the firm’s characteristics.

- This helps address one shortcoming of historical betas, which is their slow reactivity to firm’s attributes changes.

- More generally, this approach can be used by asset managers to implement portfolios more consistent with their active views on factor returns (or lack thereof).
- Portfolio Risk & Performance Analysis with Factors versus Attributes

- From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

- Targeting Market Neutrality with Fundamental versus Historical Betas
A One-Factor Model with Fundamental Alpha and Beta

- Hoechle, Schmid and Zimmermann (2015) introduce a "Generalized Calendar Time" model, which allows to represent the alpha and the beta(s) of a stock as functions of its characteristics, and whose estimation does not require sorting of stocks.

- Consider the following version of the CAPM, in which the alpha and the beta are functions of the 3 observable characteristics that define the Fama-French-Carhart factors:
  - Market capitalization;
  - B/M ratio;
  - Past 1-year return.

\[
\begin{align*}
\alpha_{it} &= \theta_{\alpha,0} + \theta_{\alpha,Cap} Cap_{it} + \theta_{\alpha,Bmk} Bmk_{it} + \theta_{\alpha,Ret} Ret_{it} \\
\beta_{it} &= \theta_{\beta,0} + \theta_{\beta,Cap} Cap_{it} + \theta_{\beta,Bmk} Bmk_{it} + \theta_{\beta,Ret} Ret_{it}
\end{align*}
\]

\[
\begin{align*}
\alpha_{it} &= \theta_{\alpha,0} + \theta_{\alpha,Cap} Cap_{it} + \theta_{\alpha,Bmk} Bmk_{it} + \theta_{\alpha,Ret} Ret_{it} \\
\beta_{it} &= \theta_{\beta,0} + \theta_{\beta,Cap} Cap_{it} + \theta_{\beta,Bmk} Bmk_{it} + \theta_{\beta,Ret} Ret_{it}
\end{align*}
\]

for stock i at date t
From Individual Stocks to Portfolios

- The fundamental beta of a stock is a measure of the market exposure over the next period conditional on the characteristics at date $t$.

- The conditional beta of a portfolio is linear in returns, so it is the weighted sum of fundamental betas:

$$
\beta_{Pt} = \sum_i w_{it} \beta_{it}
$$

$$
= \theta_{\beta,0} + \theta_{\beta,Cap} \sum_i w_{it} Cap_{it} + \theta_{\beta,Bmk} \sum_i w_{it} Bmk_{it} + \theta_{\beta,Ret} \sum_i w_{it} Ret_{it}
$$

where $Cap_{Pt}$, $Bmk_{Pt}$ and $Ret_{Pt}$ define the size, B/M and momentum scores of the portfolio.

- This method is "holding based": it requires knowledge of portfolio composition and weights, in addition to the constituents' characteristics and the model coefficients.
Revisiting the Decomposition of Performance and Volatility

- In the fundamental CAPM, the expected return and variance of a portfolio conditional on the constituents' characteristics are:

\[
E_t[r_{P,t+1}] = \alpha_{Pt} + \beta_{Pt} \times \Lambda_{MKT}
\]

\[
\text{Var}_t[r_{P,t+1}] = \beta^2_{Pt} \times \sigma^2_{MKT} + \sigma^2_{\varepsilon}
\]

- The alpha and the market contributions can be further split into 4 terms (intercept and 3 characteristics):

\[
\beta_{Pt} \Lambda_{MKT} = \theta_{\beta,0} \Lambda_{MKT} + \theta_{\beta,\text{Cap}} \text{Cap}_{Pt} \Lambda_{MKT} + \cdots
\]

\[
\beta^2_{Pt} \sigma^2_{MKT} = \theta_{\beta,0} \left[ \theta_{\beta,0} + \theta_{\beta,\text{Cap}} \text{Cap}_{Pt} + \theta_{\beta,\text{Bmk}} \text{Bmk}_{Pt} + \theta_{\beta,\text{Ret}} \text{Ret}_{Pt} \right] \sigma^2_{MKT}
\]

\[
+ \theta_{\beta,\text{Cap}} \text{Cap}_{Pt} \left[ \theta_{\beta,0} + \theta_{\beta,\text{Cap}} \text{Cap}_{Pt} + \theta_{\beta,\text{Bmk}} \text{Bmk}_{Pt} + \theta_{\beta,\text{Ret}} \text{Ret}_{Pt} \right] \sigma^2_{MKT} + \cdots
\]
Estimating the Model

- Because raw characteristics have different units and ranges, they are first converted to z-scores.

- The 8 unknown coefficients can be estimated by a pooled regression over all stocks and all periods:

\[
\sum_{i,t} \hat{u}^2_{it} \rightarrow \begin{cases} 
\hat{\theta}_{\alpha,0}, \hat{\theta}_{\alpha,Cap}, \hat{\theta}_{\alpha,Bmk}, \hat{\theta}_{\alpha,Ret} \\
\hat{\theta}_{\beta,0}, \hat{\theta}_{\beta,Cap}, \hat{\theta}_{\beta,Bmk}, \hat{\theta}_{\beta,Ret}
\end{cases}
\]

- The model is estimated over the S&P 500 universe and the period 2002-2015 (51 quarterly returns).
**Intercept Estimates**

- Because z-scores are centered, the fundamental alpha and beta of an **EW portfolio of all stocks** are constant and equal to $\theta_{\alpha,0}$ and $\theta_{\beta,0}$.

$$
\beta_{Pt} = \theta_{\beta,0} + \theta_{\beta,\text{Cap}} \sum_i \frac{1}{N} \times \text{Cap}_{it} + \theta_{\beta,\text{Bmk}} \sum_i \frac{1}{N} \times \text{Bmk}_{it} + \theta_{\beta,\text{Ret}} \sum_i \frac{1}{N} \times \text{Ret}_{it}
$$

$$
= \theta_{\beta,0}
$$

- We verify that these estimates are close to the in-sample alpha and beta of the broad EW portfolio.

<table>
<thead>
<tr>
<th>Abnormal Return</th>
<th>Market Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient $\theta_{\alpha,0}$</td>
<td>In-sample $\alpha$</td>
</tr>
<tr>
<td>0.039</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Data description: The in-sample alpha and beta of the ERI Scientific Beta US equally-weighted portfolio are estimated by regressing quarterly index returns on market returns from Ken French's library over the period 2002-2015. The coefficients $\theta_{\alpha,}$ and $\theta_{\beta,}$ are obtained by a pooled regression of the 500 stock returns.
The fundamental beta is decreasing in market cap and increasing in B/M, so the "fundamental CAPM" can explain, at least partially, the size and the value effects in returns.
Past winners are more exposed to market risk than past losers. Over the period 2002-2015, the market premium was positive, so the model predicts a negative momentum premium, which was indeed observed.
For this stock, the fundamental beta moves around the in-sample beta, but the mean of the rolling-window beta is larger than the IS beta.

Data description: The market beta of the stock is estimated by three methods: the GCT regression model ("fundamental" approach); a linear regression on the period 2002-2015 ("historical" beta); a 5-year rolling window regression. Market returns are downloaded from Ken French’s library. All returns are quarterly.
Variants of the Model

- The GCT regression model of Hoechle et al. (2015) can handle any set of discrete or continuous characteristics.

- Variant 1:
  Use other continuous characteristics (liquidity, profitability, dividend yield, etc.)

- Variant 2:
  Use discrete characteristics, e.g. sector or country. Hoechle et al. (2015) show that this is equivalent to the usual portfolio sort approach: if the alpha and the beta of a stock are functions of the sector only, they are identical to those of portfolios sorted on sector.

\[
 r_{it} = \sum_{k=1}^{10} \theta_{\alpha,k} \times S_{ki} + \left[ \sum_{k=1}^{10} \theta_{\beta,k} \times S_{ki} \right] MKT_t + u_{it} \quad \text{for stock } i \text{ at date } t
\]
Variants of the Model (Con't)

- Variant 3:
  Mix continuous and discrete characteristics, e.g. market capitalization and sectors.

\[
r_{it} = \left[ \theta_{\alpha,\text{Cap}} \text{Cap}_{it} + \sum_{k=1}^{10} \theta_{\alpha,k} S_{ki} \right] + \left[ \theta_{\beta,\text{Cap}} \text{Cap}_{it} + \sum_{k=1}^{10} \theta_{\beta,k} S_{ki} \right] \text{MKT}_t + u_{it} \quad \text{for stock } i \text{ at date } t
\]

- Variant 4:
  Allow for cross-sectional dependency in effects of characteristics on alpha and beta (more on this later).

- All of these models can be estimated by pooled regression analysis.
A More Flexible Model

- We relax the constraint of having the same coefficients $\theta$ for all stocks.

\[
\beta_{it} = \theta_{\beta,0} + \theta_{\beta,\text{Cap}} \text{Cap}_{it} + \theta_{\beta,\text{Bmk}} Bmk_{it} + \theta_{\beta,\text{Ret}} Ret_{it}
\]

\[
\beta_{it} = \theta_{\beta,0,i} + \theta_{\beta,\text{Cap},i} \text{Cap}_{it} + \theta_{\beta,\text{Bmk},i} Bmk_{it} + \theta_{\beta,\text{Ret},i} Ret_{it}
\]

- Consequences:
  - One restriction is relaxed, which reduces misspecification risk;
  - For $N$ stocks, the model has $8N$ coefficients to estimate instead of 8, which may cause loss of robustness;
  - Because the coefficients are independent from one stock to the other, the pooled regression is equivalent to $N$ time series regressions:

\[
\text{minimize } \sum_{i,t} u_{it}^2 \quad \text{equivalent to} \quad \text{minimize } \sum_t u_{it}^2 \text{ for each } i
\]
Estimated Sensitivities of Market Beta to Characteristics

- The dispersion in estimates indicates that the model with uniform coefficients imposes too much structure.
- Portfolio Risk & Performance Analysis with Factors versus Attributes
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Market Neutrality with Fundamental Vs. Historical Beta

- The traditional approach to estimating a time-varying beta is to run rolling-window regressions, but changes in firm characteristics are likely to be slowly incorporated.

- We compare the fundamental and the historical betas by constructing market-neutral portfolios based on the two methods. Each portfolio is a maximum deconcentration (best proxy for equally weighted) subject to the constraint $\beta_{portfolio} = 1$.

- To avoid look-ahead bias, coefficients of fundamental beta are estimated over a 5-year rolling window of quarterly data. Historical beta is estimated over the same sample.

- The procedure is repeated for 1,000 random universes of 30 stocks picked among the 218 that remained in the S&P 500 universe between 2002 and 2015.
Assessing Ex-Post Market Neutrality

- Over the 13-year period, the neutral portfolios constructed with the fundamental approach are closer to the market index, both in terms of beta and correlation.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St. Dev.</td>
</tr>
<tr>
<td>Historical</td>
<td>0.869</td>
<td>0.032</td>
</tr>
<tr>
<td>Fundamental</td>
<td>0.925</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Data description: 1,000 portfolios maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. They are rebalanced every quarter. The control regression is done using quarterly returns over the period 2002-2015. The 30 stocks are picked among the 218 that remained in the ERI Scientific Beta US universe for the period 2002-2015.

- Is this result robust in sub-periods?
Assessing Market Neutrality in Sub-Periods

- In some 5-year periods, the portfolio based on historical betas can largely deviate from neutrality. The fundamental one remains closer to the target.
Worst Deviations from Neutrality

- Even in the worst case, the out-of-sample beta of the fundamental portfolio remains closer to 1 than that of the historical portfolio.

![Graph showing largest distance to 1 across 1,000 universes for fundamental neutral portfolios and historical neutral portfolios.](image-url)
Conclusion:
Multi-Factor Models and Characteristics-Based Models

- The empirical link between certain characteristics and average returns can always be explained by introducing new ad-hoc factors in an asset pricing model.

- An alternative approach is to stay with a limited number of risk factors and treat characteristics as variables that determine factor exposures.

- Such models have two main advantages:
  - Theoretical: they can capture some empirical regularities (e.g., size and value effects) without the help of additional factors;
  - Practical: fundamental beta immediately responds to changes in a stock's attributes, which allows to assess the impact of a change in the portfolio composition on the factor exposure.

- The two approaches are not exclusive, and the true (still unknown) asset pricing model is likely to be multi-factor with betas depending on state variables.