Diversified or Concentrated Factor Tilts?

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At the outset, smart beta was conceived as a response to two drawbacks of broad-market, capitalization-weighted (hereafter, market-cap) indices. The first drawback is that such portfolios typically provide limited access to long-term rewarded risk factors, such as size or value, among others. The second problem is that these portfolios do not efficiently diversify unrewarded risks due to excessive concentration in the stocks with the largest market caps. Several studies have proposed methods to design indices with improved diversification as an answer to this problem. However, in recent years, the question of diversification has taken a back seat to the question of appropriate factor tilts, which has become smart-beta providers’ prime concern.

Dealing with the question of obtaining the right factor exposures gives rise to a consensus, because it provides space for active managers who had little latitude in a framework of smart-beta offerings purely focused on improving diversification. Factor investing has become an opportunity to sell stock-picking approaches as systematic strategies. Factor investing assumes that the link between stock returns and stock characteristics drives strategy performance. A factor model postulates that expected stock returns are related to factor exposures, and factor investing essentially aims at capturing exposures that will lead to higher long-term returns. Factor investing thus poses the problem of estimating expected returns through factor exposures. Ultimately, return estimation will be sensitive to both the time period and the selection of criteria used for factor identification.

In this article, we conduct a detailed comparison of the performance and risks of concentrated and diversified factor-tilted indices for six factor tilts in both the long and short terms. We discuss the conceptual issues with linking factor exposures to expected returns and argue in favor of diversifying factor-tilted portfolios. Then, using long-term U.S. stock data, we empirically compare the performance of concentrated and diversified factor-tilted portfolios on broad and narrow factor-filtered stock universes.

Related Literature and Conceptual Considerations

Factor Investing and Expected Return Estimation

In this subsection, we review the literature on the link between factor exposures and returns. We emphasize that this link is 1) based on very long-term periods and not likely to hold consistently for any short horizon and 2) subject to considerable estimation risk, especially at the stock level.
An important idea behind factor investing is that portfolios that tilt toward a range of well-documented factors have been rewarded with higher returns. However, it is well known that expected returns are notoriously hard to estimate (see Merton [1980]). We should also note with Black [1993] that “we need decades of data for accurate estimates of average expected return. We need such a long period to estimate the average that we have little hope of seeing changes in expected returns.” Thus, introducing proprietary factor definitions or concentrating on stock with the strongest exposure to factors that deliver the highest expected return is a risky business, as far as out-of-sample data reliability is concerned.

Moreover, estimating returns at the individual stock level is likely to lead to a large amount of noise. Black [1993] emphasized that expected returns cannot be reliably estimated for individual stocks. For this reason, studies that document factor premia (such as Fama and French [1993], among many others too numerous to cite) rely on portfolio-sorting approaches. Rather than trying to determine differences in returns between individual stocks, researchers have created groups of stocks and tested broad differences in returns across these. Therefore, when considering whether to create a concentrated portfolio or a diversified one for a given factor tilt, we should keep in mind that the more concentrated the portfolio becomes, the more it relies on the existence of a detailed and strict relationship between stock returns and factor exposure. If we recognize that there is a lot of noise in estimating the link between returns and stock-level characteristics, the standard way to address this issue is to make broad distinctions between groups of stocks. The assumption that a broadly diversified portfolio of many different, low-valuation stocks has a higher long-term expected return than a broadly diversified portfolio of high-valuation stocks is quite different from the assumption that the return of any individual stock with a given valuation ratio will strictly be higher than the return of any stock with a higher valuation ratio. If we believe that valuation characteristics provide an exact and deterministic link for stock returns, we would strive to build the most concentrated portfolios, using the “right” stocks. If we believe that valuation characteristics let us distinguish between differences in returns that hold (on average) across many stocks, we would strive to build well-diversified portfolios with the desired valuation tilt.

Moreover, if we believe that we can estimate the link between factor exposure and expected returns with a high degree of accuracy, we would tend to use the variable that appears to be associated with the highest returns. However, if we are agnostic about our capacity to precisely link returns to stock characteristics, we would tend to favor parsimonious and well-documented variables that have stood the test of time.

OVERVIEW OF EMPIRICAL TESTS OF FACTOR PREMIA

Rational factor investing does not rely on finding underpriced stocks but rather seeks to identify factors that lead to systematic risks that investors are unwilling to bear without a commensurate reward. It does not require an ability to pick stocks by being better than the market at processing information. Rather, it tries to identify risk factors with strong empirical evidence in favor of a positive risk premium, relative to the broad market.

Exhibit 1 summarizes the seminal literature on empirical tests of the six factors we consider in our study: size, value, momentum, low risk, profitability, and investment. We report the main findings from the following seven papers: Fama and French [1992], Jegadeesh and Titman [1993], Ang et al. [2006], Frazzini and Pedersen [2014], Fama and French [2015], Hou, Xue, and Zhang [2015a], and Novy-Marx [2013]. Panel A discusses their methodology and underlying data. Panel B provides the main results and their relevance.

The literature on factor premia highlights the existence of statistically and economically significant premia over the long term, but it is particularly interested in neither the variability in premium significance over time nor how risk-adjusted performance varies in the short term, despite being based on factor portfolios that are reconstituted at least annually.

However, the short- and medium-term dynamics of these premia are an important practical issue for investors, even when the investment horizon is long term, as they remain subject to short- and medium-term reporting obligations. Managing the risks of factor investing presupposes recognizing them, and it is clear that the reference literature has not really addressed the issue. The literature’s focus has been on explaining long-term differences in stocks’ expected returns. The recognition of these risks justifies evaluating various methods of imple-
## Exhibit 1

### Summary of Empirical Tests on Risk Factor Premia

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Factor Proxy</th>
<th>Sample</th>
<th>Portfolio Construction and Rebalancing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ang, Hodrick, Xing, and Zhang [2006]</td>
<td>WML/UMD: Returns in the past 1, 2, 3, or 4 quarters (with or without last week).</td>
<td>All firms on NYSE and AMEX (1965–1989), 1927–1964 for NYSE sub-study.</td>
<td>Various sorting, waiting, and holding periods tested. Focus is on 1 month holding of portfolios sorted on previous month’s volatility. Overlapping portfolios are rebalanced monthly. Monthly returns of long/short portfolio of value-weighted bottom/top-quintile securities.</td>
</tr>
<tr>
<td>Fama and French [2015]</td>
<td>BAB: Shrunken estimates of market beta (with separate estimation of volatilities and correlations).</td>
<td>All (US) CRSP common stocks (1926–2012). All common stocks on the Xpressfeed Global database for 19 markets of the MSCI developed universe (1984/1989–2012).</td>
<td>Monthly rebalancing. Annual rebalancing. Average monthly returns. Returns are the simple average of the top-quintile portfolios for large and small stocks minus the average of the bottom-quintile portfolios for large and small stocks, portfolios are value weighted. Returns are the simple average of the bottom quintile portfolios for large and small stocks minus the average of the top-quintile portfolios for large and small stocks, portfolios are value weighted.</td>
</tr>
<tr>
<td>Hou, Xue, Zhang [2015a]</td>
<td>RMW: Operating profitability defined as annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity at the end of fiscal year.</td>
<td>All (US) CRSP common stocks for NYSE, Amex, and NASDAQ (1963 to December 2013).</td>
<td>Annual rebalancing. Monthly returns are differences between simple averages of returns across six low and six high investment portfolios (relevant intersections of sorts by size (2), profitability (3), and investment (3)), each of which is value-weighted. Monthly rebalancing. Monthly returns are differences between simple averages of returns across six high and six low profitability portfolios (relevant intersections of sorts by size (2), investment (3), and profitability (3)), each of which is value weighted.</td>
</tr>
</tbody>
</table>
menting the factor investment paradigm and provides an angle to better appreciate the relevance of opposing approaches to factor investing: concentrated versus diversified approaches. Smart-beta providers often refer to the evidence of long-term risk premia in the academic literature to justify their approaches to factor investing but neglect the medium- and short-term risks.

THE NEED FOR DIVERSIFICATION WITHIN FACTOR-TILTED PORTFOLIOS

We review the philosophy behind diversified factor tilts. Positive exposure to rewarded factors is obviously a strong and useful contributor to expected returns. However, we must be careful to avoid the pitfalls described previously: short-term variations in long-term rewarded factors, fragility of the link to expected returns, and possible data-mining risks. A way to limit risks with factor-based strategies is to draw on a portfolio property that is a more robust source of improved risk-adjusted returns: diversification. Diversification approaches rely on estimates of risk parameters more than on estimates of expected return parameters. In fact, most diversification approaches used in practice are purely risk-based portfolio constructions that do not require any direct inputs of expected returns.

Products that aim to capture explicit risk-factor tilts through concentrated portfolios effectively neglect adequate diversification. This is a serious issue because diversification has been described as the only free lunch in finance. Diversification lets investors capture a given exposure with the lowest level of total risk, as it eliminates non-systematic risk. In contrast, taking on factor exposures exposes investors to systematic risk factors. Rewards for doing so do not constitute a free lunch. They are compensation for risk in the form of systematic factor exposures. Capturing risk premia associated with systematic factors may be attractive for investors who can accept the systematic risk exposure in return for commensurate compensation.
However, very concentrated factor-tilted strategies may also take on other, unrewarded risks. Unrewarded risks come in the form of idiosyncratic (i.e., firm-level) risk, as well as other unrewarded risks (e.g., currency risk, sector risks, and other unrewarded micro or macroeconomic factors). Financial theory does not provide any reason that such risk should be rewarded. Therefore, a sensible approach to factor investing should not only look to obtaining a factor tilt but also to achieving proper diversification within that factor tilt. To illustrate this point, we focus on the value factor, but the discussion carries over to other factors.

In fact, if the objective were to obtain the most pronounced value tilt, for example, the only unleveraged long-only strategy that would achieve this goal would be holding 100% in the single stock with the largest value tilt, as measured (for example) by its book-to-market ratio or estimated sensitivity to the value factor.

This thought experiment clearly shows that the objective of maximizing factor tilt strength is not reasonable. Moreover, this extreme case of a strong factor tilt indicates the potential issues with highly concentrated factor indices. Even if the appropriateness of such an extreme approach had been established, any value premium so captured would necessarily come with a large amount of idiosyncratic risk. This risk is not rewarded, and therefore, we should not expect the strategy to lead to an attractive risk-adjusted return. Additionally, it is unlikely that the same stock will persistently have the highest value exposure within a given investment universe. Therefore, a periodically rebalanced factor index with such an extreme concentration level will likely generate 100% one-way turnover at each rebalancing date, as the stock that the strategy held previously is replaced with a new stock that displays the highest value exposure at the rebalancing date. Although the practical implementations of concentrated factor-tilted indices will be less extreme than this example, we can expect problems with high levels of both idiosyncratic risk and turnover whenever index construction focuses too much on concentration and pays too little attention to diversification.

Interestingly, Benjamin Graham himself outlined the importance of diversification for a given factor tilt many decades ago. As noted by Asness et al. [2015], the 1973 edition of Graham’s famous book on value investing reads: “In the investor’s list of common stocks there are bound to be some that prove disappointing… But the diversified list itself, based on the above principles of selection … should perform well enough across the years. At least, long experience tells us so.” Aiming at a highly concentrated value portfolio would be completely inconsistent not only with financial theory but also with the principles put forth by the early advocates of value investing.

Cap-weighted portfolios of value stock selections may at first seem to be a more neutral implementation than equally weighted portfolios. However, it is well known that cap-weighting has a tendency to lead to very high concentration, given the heavy-tailed nature of market-cap distribution across stocks. It is well documented in the academic literature that simple cap-weighted, value-tilted portfolios have not led to attractive performance. In fact, across different studies on equity risk factors, Fama and French emphasized the need for a well-diversified portfolio as a proxy for a factor tilt. For example, Fama and French [2012] said that they “ensure that [they] have lots of stocks in each [factor-tilted] portfolio” and argue that factor-tilted portfolios should be well diversified in order to obtain factor tilts that are reliable, in the sense that factor exposures can be estimated with precision.

Hou, Xue, and Zhang [2015b] explicitly recognized the need for diversification, recalling that “value-weighted portfolio returns can be dominated by a few big stocks.” (See also Fama and French [2015].) Factor construction reflects this need for diversification. Fama and French [2012] defined the size factor as “the difference between the returns on diversified portfolios of small stocks and big stocks” and the value factor as “the difference between the returns on diversified portfolios of high book-to-market (value) stocks and low book-to-market (growth) stocks.” Thus, the value and size factors are not based on concentrated portfolios with the maximum factor-related characteristic.

To achieve diversification, the standard Fama and French value factor includes a broad selection of stocks and uses a two-tiered weighting approach. In particular, the value factor is an equally weighted combination of sub-portfolios for different market-cap ranges, effectively overweighting smaller stocks and increasing the effective number of stocks. The fact that the most widely cited research documenting the value factor’s relevance does not use simple cap-weighted factors, but rather constructs more balanced portfolios, shows the lack of academic support for industry practices using simple cap-weighted factor indices.
Moreover, several authors adopt equally weighted (EW) approaches to factor portfolios. Asparouhova, Bessembinder, and Kalcheva [2013] reviewed the literature and summarize that “examining papers published in only two premier outlets, The Journal of Finance and The Journal of Financial Economics, over a recent five-year (2005 to 2009) interval, we are able to identify 24 papers that report EW mean returns and compare them across portfolios.” (Also see Plyakha, Uppal, and Vilkov [2014].) As a recent example, Hou, Xue, and Zhang [2015b] addressed the diversification issue by forming factor portfolios that equally weight their component stocks, while excluding the smallest stocks due to implementation concerns.

Overall, it appears that the approach that proposes to construct concentrated factor indices is supported neither by the academic literature nor by common sense. On the contrary, there is a strong theoretical motivation for constructing well-diversified factor-tilted portfolios. Cochrane [1999] emphasized that any portfolio should be constructed so as to provide the efficient risk-return trade-off, in a mean–variance sense, at a given level of factor exposure. Fama [1996] showed that rewarded factors can be understood as multifactor mean–variance-efficient portfolios themselves. Thus no form of factor investing can neglect diversification as a method to construct portfolios that are not merely exposed to a given factor but also are efficient.

**PERFORMANCE OF CONCENTRATED VERSUS DIVERSIFIED TILTED PORTFOLIOS**

**Data and Methodology**

In this section, we compare portfolios for six factor tilts, each with different stock selection filters that are constructed using two different weighting schemes: cap weighting (CW) and equal weighting (EW). We use two grades of filters: the broad filtering selects the top 50% of stocks, in terms of factor scores, from the stock universe at each rebalancing, and the narrow filtering selects the top 20% of stocks. We assign all stocks factor scores that are determined by their fundamental stock characteristics or past returns. We thus assign each stock six factor scores. To construct factor-tilted portfolios, we select the top 50% or top 20% stocks by their factor scores at each annual rebalancing. This means that we choose approximately 250 and 100 stocks from the broad universe of 500 U.S. large-cap stocks. The factor-tilted cap-weighted portfolio weights the selected stocks in the proportion of their total market capitalization. The equally weighted portfolio weights the selected stocks in equal dollar proportions.

We rebalance all factor-tilted portfolios annually on the third Friday in June. The analytics on U.S. portfolios in subsequent sections use 40 years of weekly total returns, that is, returns with dividends reinvested. Stock-level data for portfolio construction and portfolio valuation come from CRSP and WRDS. The long/short factor returns used for regression come from Kenneth French’s data library.

Exhibit 2 shows detailed performance comparisons between heavily concentrated (CW) portfolios and diversified (EW) portfolios. In the long run, on average across the six factor tilts, the 50% CW portfolio’s risk-adjusted performance is inferior to that of the 50% EW portfolio. The average Sharpe ratio goes up from 0.55 to 0.66 when we move from CW to EW portfolios. Similarly with the narrow stock selection, we see an improvement in Sharpe ratio from 0.58 to 0.67 by using EW over CW.

Moving from CW to EW indubitably increases the tracking error with respect to the broad cap-weighted benchmark. But the risk-adjusted outperformance, or information ratio, remains higher for EW factor-tilted portfolios. For the 50% selection case, on average, the information ratio of EW factor-tilted portfolios is 0.66, compared with a mere 0.33 for CW factor-tilted portfolios.

We also report the historical probability of outperformance over a three-year investment horizon, defining this as the empirical frequency of outperforming the cap-weighted reference index over a given investment horizon. It is an intuitive and relevant measure for investment practitioners and shows how often and consistently the strategy has outperformed the cap-weighted reference index in the past for all possible entry points. The probability of outperformance over this medium investment horizon provides an appreciation of the stability across time of a strategy’s outperformance. The outperformance probability of EW factor-tilted portfolios is higher than that of their CW counterparts. For example, for the 50% stock selection, the outperformance probability of the EW factor-tilted portfolios is 76% on average, compared with 68% for the CW factor-tilted portfolios.

Increasing factor concentration by selecting the top 20% stocks in terms of factor scores, instead of the top 50% stocks, does not have a meaningful effect on risk-adjusted returns. For a given weighting scheme,
increasing concentration through more stringent stock selection (i.e., going from 50% to 20%) leaves the average Sharpe ratios and information ratios at relatively similar levels overall. There is no added value, from a risk-adjusted performance perspective, in having more-concentrated factor-tilted portfolios. Note that factor-tilted portfolios' average performance and Sharpe ratio, irrespective of weighting scheme (CW or EW), remain higher than those of the broad CW index, showing that the six chosen risk factors do earn (on average) a positive risk premium in the long run.

To have a deeper understanding of performance, we study the time-varying average relative performance, volatility, and tracking error, shown in Exhibit 3. The graphs are plotted using a rolling time window of three years’ length and a step size of one month. By definition, risk-factor exposure means experiencing losses or drawdowns in bad times in exchange for better-than-average market performance in the long term. The relative performance of EW and CW-tilted portfolios clearly represent the periods in which factor risk is rewarded and factor exposure leads to losses. However, the CW factor-tilted indices to some extent miss out on the opportunity to benefit from factor premia, as evidenced by much lower levels of outperformance over most periods. A visual inspection of the graphs also shows that CW and EW portfolios’ volatility and tracking error are almost the same, but their respective relative returns differ by a wide margin. In comparing the left and right sides of the Exhibit 3 graph of tracking error, we can directly compare the 50% and 20% selection-based portfolios and see that increasing stock selection severity also increases tracking error. Concentrating in fewer stocks increases the tracking error by a large amount. The rolling three-year tracking error for 20% stock selection portfolios can reach (on average) 13% to 14%. Therefore, holding a factor-tilted portfolio that contains very few stocks may also present a disadvantage for investors who face tracking-error constraints.

Exhibit 3 showed that for the same level of total risk, measured by portfolio volatility, the CW factor-tilted portfolios posted lower returns than did the EW factor-tilted portfolios, so much of the risk CW portfolios take must be unrewarded or idiosyncratic in nature. Additional analysis of performance statistics over shorter time periods suggests that diversified approaches to factor indices consistently provide stronger performance than...
EXHIBIT 3
Rolling Window Excess Returns, Volatility, and Tracking Error of Cap-Weighted and Equal-Weighted Factor Indices

<table>
<thead>
<tr>
<th>50% Stocks Selection</th>
<th>20% Stocks Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3Y-Rolling Excess Return</strong></td>
<td><strong>3Y-Rolling Excess Return</strong></td>
</tr>
<tr>
<td>Cap Weighted</td>
<td>Equal Weighted</td>
</tr>
<tr>
<td>Cap Weighted</td>
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<td>Cap Weighted</td>
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<td>Cap Weighted</td>
<td>Equal Weighted</td>
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<tr>
<td>Cap Weighted</td>
<td>Equal Weighted</td>
</tr>
</tbody>
</table>

**Notes:** The analyzed time period is 40 years: December 31, 1974, to December 31, 2014. All figures reported are average figures across six factors: size, momentum, low volatility, value, low investment, and high profitability. All factor-tilted portfolios are rebalanced annually on the third Friday in June. The analysis is done using weekly total returns (dividends reinvested) in U.S. dollars. The portfolios are constructed using a U.S. stock universe that contains the 500 largest stocks by total market cap. The market-cap-weighted index of these 500 stocks is the benchmark. All risk and return statistics are annualized and computed using a rolling window of three years (the length) and a step size of one month.

**Sources:** CRSP and WRDS.
do concentrated approaches, in particular by avoiding short-term risk during market downturns.\(^3\)

We analyze the idiosyncratic risk of concentrated and diversified factor portfolios to show that the results in Exhibit 3 are consistent with portfolio theory (i.e., more concentration leads to more idiosyncratic risk, which is unrewarded).

In order to strip out the systematic component of portfolio risk, we use a Carhart four-factor regression model (Carhart [1997]):

\[
R_i(t) - RF(t) = \alpha_i + \beta_{i}^{RM} [RM(t) - RF(t)] + \beta_{i}^{SMB} SMB(t) + \beta_{i}^{HML} HML(t) + \beta_{i}^{UMD} UMD(t) + \varepsilon_i(t) \tag{1}
\]

\(R_i(t)\) is the returns of portfolio \(i\), \(RF(t)\) is the risk free rate, \(RM(t)\) is market returns, and \(SMB(t), HML(t), UMD(t)\) are long/short factor returns.\(^4\) We then report the standard deviation of the residual returns (\(\varepsilon_i(t)\)), which is the residual risk, its interquartile range, regression alpha, and the annualized alpha per unit of residual standard deviation. The use of such a multi-factor model captures any additional factor exposures introduced by equally weighting stocks, compared with cap-weighting them. Indeed, the model captures implicit tilts of equally weighted portfolios to factors such as size, and we then compare residual performance and risk statistics across equally weighted and cap-weighted portfolios.

Another way to quantify the risk reduction achieved by diversification is to compare the portfolio’s volatility with the volatility of its factor benchmark that has been leveraged to match its returns. The factor benchmark is a synthetic portfolio with Carhart betas that are the same as those of the factor-tilted portfolio. Mathematically, it can be constructed as follows:

\[
RB_i(t) - RF(t) = \beta_i^{RM} [RM(t) - RF(t)] + \beta_i^{SMB} SMB(t) + \beta_i^{HML} HML(t) + \beta_i^{UMD} UMD(t)
\]

\[\Delta Vol_i = \left[ \frac{Vol(RB_i(t))}{Vol(RF(t))} \right] \times \left[ \frac{Vol(RF(t))}{Vol(RB_i(t))} \right] - Vol(R_i(t)) \tag{3}\]

The betas on the right-hand side of Equation (2) are obtained from the regression of portfolio \(i\) in Equation (1), and therefore \(RB_i(t)\) is the returns of the factor benchmark of portfolio \(i\). Because the magnitude of systematic risk is identical across the portfolio and its factor benchmark, the difference in volatility for the same return level can only be explained by diversification, or in other words, the reduction of idiosyncratic risk. This excess volatility can be expressed as follows (\(Ret\) operator gives the annualized returns and \(Vol\) operator gives the annualized volatility of the time series of returns):

Exhibit 4 shows that, irrespective of the weighting scheme chosen, the residual risk is larger in the case of

| Exhibit 4 |

Diversification Effects in Cap-Weighted and Equal-Weighted Factor Indices

<table>
<thead>
<tr>
<th></th>
<th>Top 50% Stocks Selected by Factor Score</th>
<th>Top 20% Stocks Selected by Factor Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cap Weighted</td>
<td>Equal Weighted</td>
</tr>
<tr>
<td>Residual Std. Deviation</td>
<td>0.51%</td>
<td>0.61%</td>
</tr>
<tr>
<td>Interquartile Range of Residual Returns</td>
<td>0.52%</td>
<td>0.62%</td>
</tr>
<tr>
<td>Ann. Alpha</td>
<td>0.68%</td>
<td>1.42%</td>
</tr>
<tr>
<td>Ann. Alpha/Residual Std. Dev.</td>
<td>1.24%</td>
<td>2.34%</td>
</tr>
<tr>
<td>Volatility Reduction</td>
<td>1.25%</td>
<td>2.10%</td>
</tr>
</tbody>
</table>

Notes: The time period of analysis is 40 years: December 31, 1974, to December 31, 2014. All figures reported are average figures across six factors: size, momentum, low volatility, value, low investment, and high profitability. All factor-tilted portfolios are rebalanced annually on the third Friday in June. The analysis is done using weekly total returns (dividends reinvested) in U.S. dollars. The portfolios are constructed using a U.S. stock universe that contains the 500 largest stocks by total market cap. The market-cap-weighted index of these 500 stocks is the benchmark. The yield on secondary market, three-month U.S. Treasury bills is the risk-free rate. We use a Carhart four-factor model for regression and annualize the reported alpha. Volatility reduction is the difference between volatility of the leveraged factor benchmark and its respective portfolio (as described in Equation (3)). The market factor is the excess returns of the cap-weighted benchmark over the risk-free rate. The size, value, and momentum factors come from Kenneth French’s data library.

Sources: CRSP and WRDS.
narrow stock selection. The average standard deviation of regression residuals increases when we move from broad to narrow stock selection, from 0.51% to 0.82% for CW, and from 0.61% to 0.79% for EW. The interquartile range of the residuals is also higher in the case of narrow stock selection portfolios, which shows that the portfolio’s idiosyncratic risk increases as the number of stocks falls.

Annualized alpha per unit of residual standard deviation, which is a measure of unexplained idiosyncratic risk-adjusted performance, increases on average from 1.24 in the case of 50% CW factor-tilted portfolios to 2.34 in the case of 50% EW factor-tilted portfolios. Similarly, reduction in volatility with respect to the respective factor benchmark is higher for EW factor-tilted portfolios than for CW factor-tilted portfolios.

A frequent criticism of EW as a weighting scheme is that EW portfolios overweight small-cap stocks, thus posing implementation challenges because small-cap stocks are relatively less liquid. Exhibit 5 shows that switching from a 50% CW factor-tilted portfolio to EW brings fewer implementation challenges than switching to a narrower 20% stock selection while remaining cap-weighted. On average, turnover does not show any considerable increase when moving from a 50% CW factor-tilted portfolio to a 50% EW factor-tilted portfolio. The average turnover of 50% CW factor-tilted portfolios is 29.25%, while that of 50% EW portfolios is 32.58%. On the other hand, switching to 20% CW factor-tilted increases the average turnover by a high margin: from 29.25% to 48.15%.

This is an interesting finding because when applied to a full universe without a factor tilt, EW is known to increase turnover. In the case of factor-tilted indices, the turnover is mainly generated by variations in stocks’ characteristics and, thus, by changes in which stocks are selected. Therefore, the weighting scheme does not contribute much additional turnover.

Because they strongly underweight smaller stocks, cap-weighted portfolios exhibit low days-to-trade (DTT) numbers. DTT is an indicator of the time required to trade the least-liquid positions in the portfolio. DTT increases when moving from CW to EW, but it remains well behaved. The increase in DTT when moving from a 50% CW portfolio to a 50% EW portfolio is similar to that of moving from a 50% CW portfolio to a 20% CW portfolio. We can use additional liquidity management rules to improve the turnover and DTT of an equally weighted strategy or that of other alternative weighting schemes. It has been shown that such implementation hurdles can be easily managed by using simple trading rules without adversely affecting portfolio performance (Amenc, Goltz, and Gonzalez [2014] and Gonzalez, Sivasubramanian, and Ye [2015]).

Overall, our results provide a strong distinction concerning the potential added value of using an improved weighting scheme for a factor-tilted portfolio versus using a more concentrated stock selection. In fact, narrowing stock selections for strong factor tilts does not add value. For a given weighting scheme, narrowing factor-based stock selection does not lead to a meaningful improvement in risk-adjusted return. As portfolios

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**Exhibit 5**
Implementation of Cap-Weighted and Equal-Weighted Factor Indices

<table>
<thead>
<tr>
<th></th>
<th>Broad</th>
<th>Top 50% Stocks Selected by Factor Score</th>
<th>Top 20% Stocks Selected by Factor Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>An. 1-Way Turnover</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cap Weighted</td>
<td>2.68%</td>
<td>29.25%</td>
<td>48.15%</td>
</tr>
<tr>
<td>Equal Weighted</td>
<td>0.20%</td>
<td>32.58%</td>
<td>48.64%</td>
</tr>
<tr>
<td>Days-to-Trade</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The time period of analysis is 40 years: December 31, 1974, to December 31, 2014. All figures reported are average figures across six factors: size, momentum, low volatility, value, low investment, and high profitability. All factor-tilted portfolios are rebalanced annually on the third Friday in June. The analysis is done using weekly total returns (dividends reinvested) in U.S. dollars. The portfolios are constructed using a U.S. stock universe that contains the 500 largest stocks by total market cap. The market-cap-weighted index of these 500 stocks is the benchmark. The reported turnover is annual one-way turnover and is averaged over 40 annual rebalancings from December 31, 1974, to December 31, 2014. A stock’s days to trade, or DTT, is the number of days required to trade the total stock position in a portfolio of $1 billion, assuming that 10% of average daily traded volume (ADTV) can be traded every day. For each portfolio, the reported DTT value is the 95th percentile of DTT values across all 10 annual rebalancings from December 31, 2004, to December 31, 2014 and across all stocks.

Sources: CRSP and WRDS.
concentrate in fewer stocks with stronger factor characteristics, average return increases, but so do volatility and tracking error. Importantly, narrower selections lead to pronounced increases in turnover and other implementation challenges. Strong factor tilts thus bring implementation challenges without providing better risk-adjusted performance. Moreover, narrower stock selections lead to increases in idiosyncratic and thus unrewarded risk.

Using a better-diversified weighting scheme such as EW improves risk-adjusted performance considerably, compared with capitalization weighting the same stock selection. This diversification creates more outperformance than concentration, as the EW portfolio built on a selection of 50% of stocks considerably outperforms the CW portfolio built on a selection of 20% of stocks. In terms of implementation, switching from cap weighting a broad (50%) stock selection to equally weighting brings fewer implementation challenges than does switching to a narrower (20%) stock selection while remaining cap weighted. In particular, turnover does not show any considerable increase when moving from CW to EW. This is an interesting finding because, when applied to a full universe without a factor tilt, EW is known to increase turnover. In the case of factor-tilted indices, the turnover is mainly generated by variations in stocks’ characteristics and thus changes to which stocks are selected. Therefore, the weighting scheme does not contribute much additional turnover. In terms of days-to-trade, EW factor-tilted indices have more challenges than do CW factor-tilted indices, but increases in days-to-trade remain well behaved. Overall, using an improved weighting scheme such as EW on a factor-tilted stock selection adds value, because risk-adjusted performance improves relative to CW while implementation challenges remain limited.

CONCLUSION

In this article, we compare different approaches factor index design. We analyze broad and narrow stock selections and two different weighting schemes, equal weighting and cap-weighting.

From a conceptual perspective, several issues arise with highly concentrated portfolios, such as cap-weighted portfolios of narrow stock selections. First, concentration in a few stocks reflects high confidence in the precision of the link between expected returns and factor exposure. We know that expected returns are notoriously difficult to estimate with precision, however, even when doing this through factor exposures. Broadly diversified factor-tilted portfolios reflect the view that we are only able to identify broad differences in expected returns across groups of stocks. Moreover, it is well known that factor premia can be identified reliably only for broadly diversified tilted portfolios. Empirical studies of factor premia insist on the necessity of constructing broad portfolios that are not unduly influenced by a small number of stocks, which has led the major studies in this area to adopt approaches that lead to diversified portfolios, notably by selecting large numbers of stocks and by using more balanced weighting approaches than simple cap weighting for the selected stocks.

Our empirical analysis confirms that concentrated factor-tilted portfolios come with problems. In fact, trying to improve the performance of cap-weighted factor-tilted portfolios by selecting fewer stocks that are most strongly tilted to the factor does not have any effect on the risk-adjusted performance. Narrow stock selections may improve returns compared with broad selections, but these increases are accompanied by higher volatility and higher tracking error, which keeps performance ratios—the Sharpe and information ratios—virtually unchanged. In addition, factor-tilted portfolios on narrow stock selections present real drawbacks, such as high idiosyncratic risk, higher turnover, and longer times to trade portfolios.

Conversely, if we focus on deconcentration by using a simple equal-weighting approach to weight stocks, we can achieve better Sharpe ratios and information ratios over both long and short investment horizons. The equally weighted portfolios incur marginally higher but manageable levels of turnover and in total do not pose implementation problems. These observations stand true across the six risk factors tested. Equally weighting stocks in a factor-tilted portfolio of course constitutes a starting point for more sophisticated diversification strategies that may help investors obtain additional diversification benefits (see Amenc et al. [2014]).

ENDNOTES

1See, for example, Choueifaty and Coignard [2008], DeMiguel et al. [2009], Maillard, Roncalli, and Teiletche [2010], and Amenc et al. [2011], among others.

2Fama and French [2012] stated: “Diversification enhances regression fits, which increases the precision of the intercepts that are the focus of the tests of competing asset pricing models.”
We use the following factor scores for each of these six factor tilts: Mid Cap, total market capitalization; Value, book-to-market (B/M) ratio (with B/M defined as the ratio of the available book value of shareholders’ equity to company market cap); Momentum, total returns over past 52 weeks, minus the last 4 weeks; Low Volatility, standard deviation of weekly stock returns over the past 104 weeks; Low Investment, past two-year total asset growth rate; High Profitability: gross profit-to-total asset ratio. The factor scores for Mid Cap, Low Volatility, and Low Investment factors are inverted. This is because, by definition, they measure their degree of being large-cap, high-volatility, and high-investment stocks respectively, and we are interested in stocks with opposite characteristics.

Weekly returns on SMB, HML, and UMD long/short factors in the U.S. can be obtained from the following web link: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

The findings hold with a great degree of consistency across the six factors. Space restrictions prevent us from listing results for each factor tilt in the main text, but these are included in the Online Appendix 1.

We compute the frequency of obtaining positive excess returns if one invests in the strategy for three years, using a rolling-window analysis with one-week step size.

We have calculated returns during the worst calendar years in terms of returns to the cap-weighted market index (see online Appendix 3). Overall, the return improvement of diversified over concentrated factor indices is particularly strong during these down markets. Moreover, we have analyzed performance and risk during non-overlapping periods of three calendar years (see Online Appendix 2). Results suggest that equally weighting leads to higher returns in 9 out of 13 sub-periods.

The regression is performed using weekly total returns. The yield on secondary market, three-month U.S. Treasury bills is the risk-free rate. The market factor is the excess return of the cap-weighted benchmark over the risk-free rate. The size (SMB), value (HML), and momentum (UMD) factors come from Kenneth French’s data library.

A stock’s days-to-trade, or DTT, is the number of days required to trade the total stock position in a portfolio of $1 billion, assuming that 10% of average daily traded volume (ADTV) can be traded every day. For each portfolio, the reported DTT value is the 95th percentile of DTT values across all 10 annual rebalancings from December 31, 2004, to December 31, 2014, and across all stocks.

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