A Stochastic Network Approach for Integrating Pension and Corporate Financial Planning

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Abstract

This paper presents a multi-period stochastic network model for integrating corporate financial and pension planning. Pension planning in the United States has gained importance with the population aging and the growth of retirement accounts. In certain cases, the pension plan assets are several times larger than the value of the company itself (e.g. General Motors – Market cap: $19 billion, Pension plan assets: $67 billion, Estimated pension fund deficit: $25 billion – in December 31, 2002; see General Motors Corporation (2003)). Thus, pension investment decisions can have a sizeable impact on a company’s long-term financial health. However, pension planning is rarely linked to general corporate planning systems since the domain falls outside traditional corporate budgeting and planning processes.

We develop a consistent framework for combining the pension plan with the corporate financial plan via a stochastic optimization model. The approach can be specialized as a stochastic network, providing possible improvements in computational efficiency and ease of understanding. The goals of the integrated planning model can be readily tailored to the company’s environment to be consistent with the existing corporate strategy. For example, there are numerous measures of risks for a large corporation, such as volatility of earnings, downside risks with respect to target earnings, share price, etc. The developed framework can be adapted to these objectives. In any event, we suggest that several risk measures be displayed to the senior managers so that they better understand the inherent tradeoffs, especially regarding temporal issues.
4 A Stochastic Network Approach for Integrating Pension and Corporate Financial Planning

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4.1 Introduction

This chapter presents a multi-period stochastic network model for integrating corporate financial and pension planning. Pension planning in the United States has gained importance with the population aging and the growth of retirement accounts. In certain cases, the pension plan assets are several times larger than the value of the company itself (e.g. General Motors – Market cap: $19 billion, Pension plan assets: $67 billion, Estimated pension fund deficit: $25 billion – in December 31, 2002; see General Motors Corporation (2003)). Thus, pension investment decisions can have a sizeable impact on a company’s long-term financial health. However, pension planning is rarely linked to general corporate planning systems since the domain falls outside traditional corporate budgeting and planning processes.

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The general topic of pension planning affects several groups. First, the company desires to minimize its contribution to the plan over time while
being able to meet its obligations to the retirees. For defined benefit plans, an annual actuarial valuation is conducted in order to assess the so-called pension surplus or deficit – market value of assets minus adjusted discounted value of liability cash-flows:

\[
\text{Surplus} = \text{Market value of assets} - \text{Discounted (liabilities)}
\]

In most cases, there are two general measures of liabilities: (1) accumulated benefit obligations (ABO), and (2) projected benefit obligations (PBO). Roughly speaking, the former represents current legal obligations (say if the plan were to close today), whereas the latter depicts an estimate of the future liabilities (as the company employees age, change positions, etc.). Generally, the PBO is greater than the ABO. Liabilities are defined by FAS 87 (Financial Accounting Standards Statement No: 87) and reported in the company’s financial statements. An important consideration involves the requirement that the company must make contributions when the plan is deemed sufficiently underfunded (as determined by ABO and sometimes PBO). The contribution decision is a major link between the core corporation and the pension plan; we model this as an arc in the stochastic network at each time period, for each scenario (more in the next section). Contributions can also be made voluntarily when the company has sufficient cash and under certain IRS regulations. We will show that this decision can be evaluated in the integrated planning model.

The employees have an important stake in the health of the pension plan. A company falling into bankruptcy will at times turn over the pension plan to the quasi-government agency – the Pension Benefit Guarantee Corporation (PBGC) to administer the plan if the company runs out of funds. This organization protect the employees, but only to a certain degree since the PBGC has limited resources and may not be able to raise benefits, for example, with inflation. Thus, a large surplus is desirable from the employees’ perspective. However, most companies would rather keep funds in the core company, all else being equal, since retained earnings are more flexible than pension funds. It is noteworthy that over the long US bull market many companies have been able to generate accounting “profits” by generating returns above the projected target returns. Recent events have shown that companies should evaluate their assets and liabilities together. Unfortunately, many have not. We will show that careful integrated planning could have largely eliminated the large decrease in surplus or the loss of surpluses.

Due to the size of the issue (almost $4 trillion in assets), pension plans impact the health of the US economy. What will happen if a number of large plans are unable to continue? The $8 billion surplus in PBGC at the beginning of 2002 had disappeared by early 2003 due to only a couple of large bankruptcies. Thus, pension plan issues have public policy implications. The area of risk management for these plans can be improved by developing more reliable and comprehensive tools.
A stochastic planning system consists of three elements: (1) a stochastic scenario generator; (2) a corporate simulator; and (3) an optimization module for discovering non-dominated recommended solutions. See Mulvey and Ziemba (1998) for a discussion of the general issues.

The first, and to some degree, the most important element involves the scenario generator. This system of stochastic equations drives the underlying stochastic processes (Mulvey (1996)). A simple representation is shown in Figure 4.1; here, we depict a scenario tree for the evolution of future uncertainties. The planning horizon is divided into $T$ time periods (generally years for pension planning). Most pension plans aim for 5 to 7 or more years at the planning horizon. Actuarial tradition requires that the cash-flows be projected over longer time periods – sometimes up to 50 or 75 years. However, the actual planning model will be designed for shorter periods. At each period, the long-term cash-flows are discounted to define the ABO or the PBO.

Importantly, a scenario is defined as a single branch from the root to any leaf of the tree. Thus, all of the parameter uncertainties are depicted along this branch. We model each scenario as a stochastic network (see Section 4.2). The overall stochastic network requires a set of additional constraints, called non-anticipativity conditions. In most cases, a set of scenarios is selected by employing variance reduction methods. We define the sample as set $\{S\}$.

The next section defines the integrated model and discusses some of the technical aspects of constructing and implementing the planning system. In Section 4.3, we describe a historical analysis of a typical pension plan, as developed by a respected actuarial firm, Towers Perrin. The purpose of these empirical tests is to illustrate several ways to employ the integrated model for corporate planning. Also, we show that the large loss of surplus that many firms experienced over the past three years (2000 to early 2003) could have been largely prevented by careful asset and liability management.

### 4.2 Multi-Period Investment Model

This section defines the integrated pension and corporate financial planning problem as a multi-stage stochastic program. The basic model is a variant of Mulvey et al. (1997). The major extension is that there is a company entity in the model in addition to the entities that constitute the pension fund. While the company grows through the planning horizon, it has to determine whether to make cash contributions to the pension plan.

First, we define the planning horizon $T$ as $T = \{1,\ldots,\tau,\tau + 1\}$. We focus on the pension plan’s position and the value of the company at the end of period $\tau$. Investment and contribution decisions occur at the beginning of each time stage.

Asset investment categories are defined by set $A = \{1,2,\ldots,I\}$, with category 1 representing cash. The remaining categories can include broad in-
vestment groupings such as stocks, long-term government or corporate bonds and foreign equity. The categories should track well-defined market segments. Ideally, the co-movements between pairs of asset returns would be relatively low so that diversification can be done across the asset categories.

As with single-period models, uncertainty is represented by a set of distinct realizations \( s \in S \). Scenarios may reveal identical values for the uncertain quantities up to a certain period - i.e., they share common information history up to that period. Scenarios that share common information must yield the same decisions up to that period. We address the representation of the information structure through non-anticipativity conditions. These constraints require that any variables sharing a common history, up to time period \( t \), must be set equal to each other. See equations (4.8).

We assume that the plan portfolio is rebalanced at the beginning of each period. Alternatively, we could simply make no transaction except to reinvest any dividend and interest – a buy and hold strategy. For convenience, we also assume that the cash flows are reinvested in the generating asset category. Another assumption is that we know the current asset allocation of the pension fund. For a variant of this model, where only the initial wealth is known (i.e. initial asset weights are decision variables), see Mulvey and Simsek (2002).

For each \( i \in A, t \in T \), and \( s \in S \), we define the following parameters and
decision variables:

**Parameters**

- \( r_{i,t}^s = 1 + \rho_{i,t}^s \), where \( \rho_{i,t}^s \) is the percentage return for asset \( i \), in time period \( t \), under scenario \( s \) (projected by a stochastic scenario generator, for example, see Mulvey et al. (2000)).
- \( g_t^s = 1 + \gamma_t^s \), where \( \gamma_t^s \) is the percentage growth rate of the company in time period \( t \), under scenario \( s \).
- \( b_t^s \) Payments to beneficiaries in period \( t \), under scenario \( s \).
- \( \pi^s \) Probability that scenario \( s \) occurs, \( \sum_{s \in S} \pi^s = 1 \).
- \( v_{i,1}^s \) Amount of money in asset category \( i \), at the beginning of period 1, under scenario \( s \), before rebalancing.
- \( z_1 \) Value of the company at the beginning of time period 1.
- \( \sigma_{i,t}^s \) Transaction costs incurred in rebalancing asset \( i \) at the beginning of period \( t \) (symmetric transaction costs are assumed, i.e. cost of selling equals cost of buying).

**Decision Variables**

- \( x_{i,t}^s \) Amount of money in asset category \( i \), at the beginning of period \( t \), under scenario \( s \), after rebalancing.
- \( v_{i,t}^s \) Amount of money in asset category \( i \), at the beginning of period \( t \), under scenario \( s \), before rebalancing.
- \( p_{i,t}^s \) Amount of asset \( i \) purchased for rebalancing in period \( t \), under scenario \( s \).
- \( d_{i,t}^s \) Amount of asset \( i \) sold for rebalancing in period \( t \), under scenario \( s \).
- \( w_t^s \) Wealth (pension plan) at the beginning of time period \( t \), under scenario \( s \).
- \( z_t^s \) Value of the company before a contribution is made in period \( t \), under scenario \( s \).
- \( y_t^s \) Value of the company after a contribution is made in period \( t \), under scenario \( s \).
- \( c_t^s \) Amount of cash contributions made in period \( t \), under scenario \( s \).

Given these definitions, we present the deterministic equivalent of the stochastic problem.

**Model (SP)**

Maximize Expected Utility = \( \sum_{s \in S} \pi^s U(w_{t+1}^s, z_t^s) \) \hspace{1cm} (4.1)

subject to:

\[ \sum_{i \in A} v_{i,t}^s = w_t^s \hspace{0.5cm} \forall s \in S, \hspace{0.2cm} t = 1, \ldots, \tau + 1, \] \hspace{1cm} (4.2)
\[
v_{i,t+1}^s = v_{i,t}^s x_{i,t}^s \quad \forall s \in S, \ t = 1, \ldots , \tau, \ i \in A, \quad (4.3)
\]
\[
z_{t+1}^s = g_t^s y_{t}^s \quad \forall s \in S, \ t = 1, \ldots , \tau, \quad (4.4)
\]
\[
y_{t}^s = z_{t}^s - c_{t}^s \quad \forall s \in S, \ t = 1, \ldots , \tau, \quad (4.5)
\]
\[
x_{i,t}^s = v_{i,t}^s + p_{i,t}^s (1 - \sigma_{i,t}) - d_{i,t}^s \quad \forall s \in S, \ i \neq 1, \ t = 1, \ldots , \tau, \quad (4.6)
\]
\[
x_{1,t}^s = v_{1,t}^s + \sum_{i \neq 1} d_{i,t}^s (1 - \sigma_{i,t}) - \sum_{i \neq 1} p_{i,t}^s - b_{t} + c_{t}^s, \quad \forall s \in S, \ t = 1, \ldots , \tau, \quad (4.7)
\]

\[
x_{i,t}^s = x_{i,t}^{s'} \text{ and } \forall s \text{ and } s' \text{ with identical past up to time } t. \quad (4.8)
\]

A generalized network investment model is presented in Figure 4.2. This graph depicts the flows across time for each of the asset categories and the company. While all constraints cannot be put into a network model, the graphical form is easy for managers to comprehend. General linear and nonlinear programs, the preferred model, are now readily available for solving the resulting problem. However, a network may have computational advantages for extremely large problems, such as security level models.

The objective function (4.1), which we aim to maximize, is the expected value of the firm’s utility function at the end of the planning horizon, denoted by \( U(w_{\tau+1}, z_{\tau+1}) \). It should be noted that this is a function of both the value of the company and the pension plan surplus at that time. This could very well be a von Neumann-Morgenstern utility function. The function should be defined very carefully because money that goes into pension funds is very difficult to extract. Although the firm will try to avoid a deficit in its pension plan as much as it can, it will also try to avoid transferring money to the plan because the company itself, not the pension fund may have financial troubles. In this chapter, we render several simplifications to illustrate the analysis more clearly. More realistic assumptions will be made for a further study.

The combined utility function is quite flexible, with several possible formulations. For instance, we might maximize a weighted combination of the net value of the pension plan (possibly discounted) and the core company’s value. Alternatively, we might constrain the pension plan value at the horizon date to be slightly positive, and then maximize the company’s value. The latter objective allows for a variety of strategies to be considered, including the benefits of voluntary contributions today (\( t = 1 \)).

As in single-period models, the nonlinear objective function (4.1) can take several different forms. If the classical return-risk function is employed, then (4.1) becomes \( \text{Max } \eta^* \text{Mean}[f(w_{\tau+1}, z_{\tau+1})] - (1 - \eta)^* \text{Risk}[f(w_{\tau+1}, z_{\tau+1})] \), where \( f(w_{\tau+1}, z_{\tau+1}) \) is a function of values of the pension fund and the company at the end of the planning horizon. The value of this function may be interpreted as the combined wealth. \( \text{Mean } \) and \( \text{Risk } \) are the expected value and the risk of combined final wealth across the scenarios at the end of period \( \tau \). Parameter \( \eta \) indicates the relative importance of risk as compared with the
expected value. This objective leads to an efficient frontier of wealth at the end of period \( \tau \) by allowing alternative values of \( \eta \) in the range \([0, 1]\). A viable alternative to mean-risk is the von Neumann-Morgenstern expected utility of wealth at the end of period \( \tau \). One might consider replacing \( w_t \) with the pension plan’s surplus, since it is a more important measure than the amount of assets in the pension fund. The surplus definition is discussed in detail towards the end of this section.

Constraint (4.2) represents the total value of assets in the fund in the beginning of period \( t \). This constraint can be modified to include assets, liabilities, and investment goals, in which case, the modified result is called the surplus wealth (Mulvey (1989)). Many investors render investment decisions without reference to their liabilities or investment goals. Mulvey (1989) employs the notion of surplus wealth to the mean-variance and the expected utility models to address liabilities in the context of asset allocation strategies. Constraint (4.3) depicts the wealth accumulated at the beginning of period \( t \) before rebalancing in asset \( i \). The growth of the company is depicted in constraint (4.4). Constraint (4.5) represents the balance constraint for the company by subtracting the cash contributions at each period. The
flow balance constraint for all assets except cash for all periods is given by constraint (4.6). This constraint guarantees that the amount invested in period $t$ equals the net wealth for the asset. Constraint (4.7) represents the flow balancing constraint for cash. Please note that the benefit payments and the cash contributions are accounted for in this constraint. Non-anticipativity constraints are represented by (4.8). These constraints ensure that the scenarios with the same past will have identical decisions up to that period. While these constraints are numerous, solution algorithms take advantage of their simple structure (Birge and Louveaux (1997), Dantzig and Infanger (1994), Kall and Wallace (1994), Mulvey and Ruszczynski (1995)).

Model (SP) depicts a split variable formulation of the stochastic asset allocation problem. This formulation has proven successful for solving the model using techniques such as the progressive hedging algorithm of Rockafellar and Wets (1991) and the DQA algorithm by Mulvey and Ruszczynski (1995). The split variable formulation can be beneficial for direct solvers that use the interior point method. The constraint structure for this formulation is depicted in Figure 4.3.

By substituting constraint (4.8) back in constraints (4.2) to (4.7), we
obtain a standard form of the stochastic allocation problem. Constraints for this formulation exhibit a dual block diagonal structure for two-stage stochastic programs and a nested structure for general multi-stage problems. This formulation may be better for some direct solvers. The standard form of the stochastic program possesses fewer decision variables than the split variable model and is the preferred structure by many researchers in the field. This model can be solved by means of decomposition methods, for example, the L-shaped method (a specialization of Benders algorithm). See Birge and Louveaux (1997), Consigli and Dempster (1998), Dantzig and Infanger (1993), Kouwenberg and Zenios (2001).


Pension plan administrators must make periodic cash (or in some cases, stock) contributions and pay benefits to the plan’s retirees. A pension plan must conduct annual valuations to determine the plan’s ability to pay its beneficiaries in the future. To this end, actuaries calculate the plan’s surplus or deficit as follows:

\[ S_{w_s}^s = w_t^s - \text{Present value}(b_{t+1}^s, b_{t+2}^s, \ldots, b_{r+2}^s) \]  

where the present value is taken over the nodes in the sub-tree emanating out of the node \((s, t)\).

Generally, a contribution is required when the plan falls into deficit or when obligations exist from previous time periods. The exact amount of the contribution depends upon actuarial rules and the structure under which the plan operates. These rules are complex formulae based on the company’s position and the existing economic environment. We define these relationships with the simple functional form as follows:

\[ c_t^s = f(S_{w_t}^s). \]  

To simplify SP, we develop a model possessing a special policy rule, called fixed mix or dynamically balanced, as a special case of (SP). Define the proportion of wealth to be: \( \lambda_{i,t}^s \) for each asset \( i \in A \), time period \( t \in T \), under scenario \( s \in S \). A dynamically balanced portfolio enforces the following condition at each juncture:

\[ \lambda_i = \frac{x_{i,t}^s}{\sum_{i \in A} x_{i,t}^s}, \]  

where \( \lambda_i = \lambda_{i,t}^s \).

This constraint ensures that the fraction of wealth in each asset category \( i \in A \) is equal to \( \lambda_i \) at the beginning of every time period. Ideally, we would maintain the target \( \lambda \) fractions at all time periods and under every scenario. Practical considerations prevent this simple rule from being implemented in a direct fashion. Adding decision rules to model (SP) gives rise to a non-convex optimization model. Thus, the search for the best solution requires specialized algorithms.
4.3 Empirical Analysis

In this section, we highlight the results of a historical analysis for a sample pension plan. This historical study is not an exact implementation of the model presented in the previous section. First of all, it is a backtesting analysis, i.e., there’s only one scenario which consists of past data. Furthermore, our goal is to show whether it could have been possible to protect the surplus of a pension fund within the integrated framework. Some of the assumptions we make here are not very realistic due to legal regulations in this area; however, they are necessary for an understandable illustration of the analysis. Most of these simplifications will be relaxed in a further study, in which we will carry out the forward-looking analysis using the scenario structure and the non-anticipativity constraints.

The problem involves strategic asset allocation and contribution decisions for a pension plan. The sample plan’s liability data is generated by Towers-Perrin. The present values of the projected benefit obligations (PBO) and actual benefit obligations (ABO) are given in Table 4.1. We use PBO values as the present value of the liabilities in a given year and monthly evaluations are obtained through interpolation. We assume that the pension fund is initially 5% over-funded (i.e., the ratio of assets to the present value of liabilities is 1.05).

It should be noted that the company we choose has an initially over-funded pension plan, which may not always be the case. We also assume that this is a stable company with a constant growth rate (more details later). We assume that the company will try to protect their pension plan against a deficit possibility, i.e., the pension fund is not allowed to be underfunded. In case there’s a chance of deficit, the company will bail out the pension fund by making cash contributions equaling the potential deficit amount. This assumption is not very realistic, due to legal standards in this area. First of all, the companies are not required to keep their pension funds over-funded at all times, and besides, the contribution rules are very complicated and a 100% contribution almost never happens. Since our initial goal is to meet the no-deficit plan without making any contributions, we will stick with this simple contribution rule.

For the pension fund, we consider portfolios composed of positions in S&P500, MSCI EAFE index, long-term (20+yr) US T-Bonds, Salomon BIG index, US Corporate AA Long bonds and Cash. The values of these indices were obtained from Datastream (2002). The collected time series involve monthly data for the period from January 1990 through December 2001.

First we analyze the statistical characteristics of these investment instruments. The results are depicted in Table 4.2.

For simplicity, we assume that the company has a constant growth rate of 1% per month and is initially valued at ten times the value of its liabilities. This constant growth assumption is equivalent to discounting the company’s value by a constant risk-adjusted discount rate.
### Table 4.1. Liability data for the sample pension plan ($)

<table>
<thead>
<tr>
<th>Year (end of)</th>
<th>PBO</th>
<th>ABO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989</td>
<td>591,532,524</td>
<td>495,963,198</td>
</tr>
<tr>
<td>1990</td>
<td>601,101,106</td>
<td>504,446,255</td>
</tr>
<tr>
<td>1991</td>
<td>684,807,026</td>
<td>572,182,478</td>
</tr>
<tr>
<td>1992</td>
<td>724,325,694</td>
<td>608,687,836</td>
</tr>
<tr>
<td>1993</td>
<td>828,201,120</td>
<td>698,724,056</td>
</tr>
<tr>
<td>1994</td>
<td>739,178,438</td>
<td>630,781,498</td>
</tr>
<tr>
<td>1995</td>
<td>922,373,874</td>
<td>781,342,986</td>
</tr>
<tr>
<td>1996</td>
<td>893,551,753</td>
<td>764,822,375</td>
</tr>
<tr>
<td>1997</td>
<td>980,837,359</td>
<td>840,299,588</td>
</tr>
<tr>
<td>1998</td>
<td>1,047,130,795</td>
<td>901,380,972</td>
</tr>
<tr>
<td>1999</td>
<td>953,045,603</td>
<td>823,637,720</td>
</tr>
<tr>
<td>2000</td>
<td>1,048,601,461</td>
<td>899,258,414</td>
</tr>
<tr>
<td>2001</td>
<td>1,130,299,827</td>
<td>969,554,542</td>
</tr>
</tbody>
</table>

### Table 4.2. Statistical characteristics of asset classes

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Annualized Arithmetic Return</th>
<th>Annualized Geometric Return</th>
<th>Annualized Risk (Stand. Dev.) of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>5.06%</td>
<td>5.18%</td>
<td>0.40%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>13.21%</td>
<td>12.85%</td>
<td>14.55%</td>
</tr>
<tr>
<td>MSCI EAFE</td>
<td>4.11%</td>
<td>2.70%</td>
<td>17.03%</td>
</tr>
<tr>
<td>US T-Bonds (20 +years)</td>
<td>9.20%</td>
<td>9.20%</td>
<td>8.59%</td>
</tr>
<tr>
<td>Salomon BIG</td>
<td>7.91%</td>
<td>8.13%</td>
<td>3.78%</td>
</tr>
<tr>
<td>US Corp AA</td>
<td>8.87%</td>
<td>9.02%</td>
<td>6.45%</td>
</tr>
<tr>
<td>Long Bonds</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The planning horizon is between January 1990 and December 2001. We employ a monthly rebalanced fixed-mix decision rule, and every month the plan’s surplus value is evaluated. For simplicity, the transaction costs are ignored. We try to optimize several objectives such as minimizing discounted sum of contributions, minimizing the volatility of plan surplus, maximizing a weighted sum of plan’s ending surplus and company’s ending value. This last objective may be interpreted as a combined ending wealth. In our analysis, we assign a unit weight to the company value and a weight of 0.8 to pension plan’s ending surplus, considering that company value matters slightly more than the plan’s surplus.

Given these assumptions, we note that we are able to meet the no-deficit requirements every month without making any contributions. Therefore, we first set all the contributions to zero level. This eliminates the company sub-tree and results in an asset-liability model for the pension plan. In the absence of transaction costs, we do not need to keep track of amounts before or after rebalancing, which means that the portfolio weights are the only decision variables. There are two conflicting objective functions in this case. The first one is maximizing the ending surplus of the pension fund and the second one is minimizing the funded ratio volatility, which is defined as the standard deviation of the difference vector of the funded ratio. Funded ratio is defined as the ratio of assets to the present value of liabilities.

The efficient frontier corresponding to the no-contribution case is depicted in Figure 4.4. In Table 4.3, the portfolio mixes corresponding to the efficient solutions are presented. The results show that without any contribution, plan
4.3 Empirical Analysis

Table 4.3. Asset allocation and objective function values for the points (Pt.) labeled on the frontier. The asset allocations are in percentages. Since there are no contributions, company’s final value is constant. Therefore, the combined ending wealth is not a separate objective.

<table>
<thead>
<tr>
<th>Pt</th>
<th>Cash</th>
<th>S&amp;P 500</th>
<th>MSCI</th>
<th>US EAFE</th>
<th>T-Bond 20+ Yr</th>
<th>Lehman Corp</th>
<th>Salomon Big</th>
<th>Surplus Ratio (billion $)</th>
<th>Volatility (billion $)</th>
<th>Combined Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>42</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>0.73</td>
<td>24.87</td>
</tr>
<tr>
<td>2</td>
<td>56</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>43</td>
<td>0</td>
<td>0</td>
<td>0.20</td>
<td>0.74</td>
<td>24.88</td>
</tr>
<tr>
<td>3</td>
<td>38</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>58</td>
<td>0</td>
<td>0</td>
<td>0.30</td>
<td>0.82</td>
<td>24.96</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>6</td>
<td>0</td>
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surplus can reach to the highest possible ending surplus through a portfolio of roughly 40% stocks and 60% bonds. This allocation is exactly the opposite of the typical pension plan asset allocation, which is roughly 60% stocks and 40% bonds.

Next, we assume that the company can make voluntary contributions to the plan, whether there’s a surplus or not. However, we set the discounted sum of all contributions to $20 million. In other words, this discounted total should be met, but the decision of making the contribution is not dependent on the funded ratio of the plan at any point. In this case, the objective of maximizing the plan’s ending surplus is replaced by maximizing the combined ending wealth, as defined previously. The resulting efficient frontier is plotted on top of the previous one and depicted in Figure 4.5. The attributes of the labeled points are given in Table 4.4. As expected, with the voluntary contribution, at any risk level, the plan’s ending surplus is higher than that of the no-contribution case. This time, the high-end of the efficient frontier gives a more typical portfolio. We are able to meet the surplus requirement across time with the help of cash contributions, and maximize the combined ending wealth with an equity-dominating portfolio. In fact, we find that relaxing the $20 million limit to $48 million results in a 100% S&P500 portfolio for
Table 4.4. Asset allocation and objective function values for the points (Pt.) labeled on the frontier. The asset allocations are in percentages. Because there are contributions, the combined wealth replaces the ending surplus as one of the objective.

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After comparing the last two columns of the two data tables; it should be noted that, for lower risk levels, making contributions improves the combined ending wealth (upward improvement in the efficient frontier). However, after a certain point, the no-contribution plan yields higher combined ending wealth values. This dominance continues until you cannot achieve a higher ending value without making any contributions. After that point, for higher-risk levels you can always gain more by making cash contributions. The efficient frontiers for this comparison are not shown, since the chart doesn’t reflect the observation very well, due to scaling differences.

Next we analyze how the bonds are distributed as we go along the efficient frontier for the case allowing contributions. In Figure 4.6, each line shows how the weights of respective bonds in the portfolio change as we go from the minimum risk portfolio to the maximum reward portfolio. It is clear that Salomon BIG, which includes shorter term bonds, is more important for a risk-averse portfolio. As maximizing the combined wealth gains more importance the weight is switched to longer term bonds. When reward-maximization is the only goal, corporate long bonds would be the best choice among all bond choices.
4.3 Empirical Analysis

Fig. 4.5. Efficient frontier for the case in which company can make voluntary contributions. Ending surplus is plotted versus the funded ratio volatility.

Fig. 4.6. Bonds and their weights in the portfolios corresponding to the labeled points on the efficient frontier (contribution is allowed).
4.4 Conclusions and Future Research

The chapter has defined an integrated pension and financial planning system by means of a stochastic network. For simplicity, we omitted a number of details for the corporate planning system. The sole requirement was to project the company’s value, as a function of the scenario set. Large financial companies, especially global insurance companies, perform this task under the title of DFA (dynamic financial analysis). A more comprehensive approach would present further details of the company’s operations within the model. The general corporate planning approach is to discount with a single risk-adjusted factor. The integrated model we present improves upon these concepts.

Solving stochastic programming problems has become practical due to the large improvements in computer hardware and software (Birge and Louveaux (1997), Kall and Wallace (1994)). There have been a number of successful implementations of stochastic planning models. The stochastic network model has certain advantages over general nonlinear programs, especially with respect to improving the model’s understandability.

The empirical results show that the integration of pension and corporate planning is not only feasible but also it can improve company performance. This improvement is particularly significant when the company’s profit is correlated with the business cycle. A large required contribution during an economic downturn can be devastating to certain companies. And in other cases, there are benefits to making voluntary contributions, as shown in the empirical tests. Additional work can be done to refine these concepts, of course. For instance, the model can be extended to address the “optimal” contribution strategy for underfunded plans. Herein, the investment policy rule will not be the fixed-mix rule. In addition, we plan to extend the model to address the problem of decentralized risk management. Many corporations do not have a centralized headquarters with timely and adequate information to carry out the integrated planning system as proposed. A practical decentralized management system is required for these firms.

References


