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Evaluating a Trend-Following Commodity Index for Multi-Period Asset Allocation

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Abstract

The Mt. Lucas index provides a systematic approach for capturing a portion of the return of trend-following commodity traders. We analyze the Mt. Lucas Index across different historical periods, evaluating its performance within a multi-period asset allocation framework. Our results indicate that the index improves the overall return/risk characteristics of the multi-period asset allocation model. We show that the total return consists of: 1) T-Bill returns on marginable assets, 2) static returns from trend-following futures markets, and 3) rebalancing gains. The importance of the third element is emphasized.

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Original Version: February 27, 2003
This Version: August 15, 2003

ABSTRACT

The Mt. Lucas index provides a systematic approach for capturing a portion of the return of trend-following commodity traders. We analyze the Mt. Lucas Index across different historical periods, evaluating its performance within a multi-period asset allocation framework. Our results indicate that the index improves the overall return/risk characteristics of the multi-period asset allocation model. We show that the total return consists of: 1) T-Bill returns on marginable assets, 2) static returns from trend-following futures markets, and 3) rebalancing gains. The importance of the third element is emphasized.

Alternative investments (hedge funds, commodity pools, private equity, venture capital, timber, and real estate) are particularly valuable when the return characteristics are driven by factors that are different than traditional equity and fixed-income investments. See, for example, Schneeweis and Spurgin [1998], Fung and Hsieh [2001], and Edwards and Caglayan [2001]. These assets provide significant diversification benefits and attractive earnings potential. However, there are several possible drawbacks – a wide dispersion of outcomes, unpredictability of performance, a reduction in liquidity, and at times difficult to judge on an ongoing basis. Schneeweis et al. [2002] discuss these issues. Recent studies (Schneeweis and Martin [2001], Georgiev [2001], Schneeweis and Spurgin [1998]) suggest that institutional investors may improve portfolio performance by employing alternative investments. Jensen et al. [2002] analyze commodity futures in the context of short-term tactical investment. Here, we analyze the use of a systematic index of commodities in a multi-period portfolio model suitable for long-term investors.

We first describe the properties of the Mt. Lucas Management (MLM) index and its use in multi-period portfolio models. Following that, we conduct an empirical study, finding that a systematically managed portfolio of futures contracts can be a useful component of a long-term investment strategy. We propose a modification to the index for long-term investors.

The MLM index was launched in 1989 as a passive benchmark for commodity futures investors and managers. The index is easy to understand and describe: It includes equally weighted investment in twenty-five liquid futures markets in seven different sectors (four-percent in each market). Exhibit 1 lists the twenty-five markets organized by sector. Importantly, at the end of each month, the index is rebalanced so that all the twenty-five components are equally weighted.

Commodity trading advisors (CTAs) often employ leverage and take short positions so any tracking index should allow for these situations. The MLM index may not exactly be a proxy for trend-following systems (Fung and Hsieh [2001]); nonetheless, it employs a simple

trend-following strategy: If the 200-day moving average of closing prices is greater than the closing price of the nearby futures contract on the penultimate trading day of the previous month then the position is short, otherwise it is long.

Currencies	Energy	Financials	Grains	Metals	Softs	Meats
British Pound	Heating Oil	Treasury Bonds	Corn	Gold	Coffee	Live Cattle
Canadian Dollar	Unleaded Gas	Five-Year notes	Soybeans	Silver	Sugar	
Euro	Crude Oil	Ten-Year Notes	Soybean Meal	Copper	Cotton	
Swiss Franc	Natural Gas		Soybean Oil			
Japanese Yen			Wheat			
Australian Dollar						

Exhibit 1: The twenty-five components of the MLM index arranged by sector. The index is rebalanced at the beginning of each period to give a four percent allocation to every component in the index. A simple trend-following rule determines whether the position is short or long.

This trend-following strategy has been studied previously. Waksman [2000] states that this strategy might capture the return to holding "price volatility" in commodity markets. In this view, trend-following strategies behave like an options straddle, providing large payoffs during periods of high volatility. See also Fung and Hsieh [2001]. Jaeger et al. [2002] list reasons that help to explain the economic rationale behind the returns of a trend-following index, suggesting that price fluctuations create supply and demand mismatches between the natural buyers and sellers of commodities futures contracts (usually producers and consumers of commodities) and that trend-following investors can benefit by these imbalances. In this sense, trend-following investors provide insurance against volatility to commodity producers and consumers via the futures market.

Jensen et al. [2002] examine the MLM index in a tactical asset allocation context, finding that the MLM index historically has higher return when US monetary policy is contractionary. In contrast, equities generally have a higher return when monetary policy is expansionary. To improve performance, they suggest tactically increasing or decreasing allocation to futures depending on the state of monetary policy. However, recent market events may not support these findings.

In this paper, we show how these properties of trend-following commodity asset classes can be usefully employed by long-term investors in a strategic fashion within a multi-period portfolio model. We will show that the MLM index performs well during periods of high volatility, not only due to the trend-following rule, but also due to their rebalancing strategies.

ASSET CLASSES

In this study, we analyze the MLM index and six traditional asset classes (monthly data from January 1979 to April 2002): 1) S&P 500, 2) Morgan Stanley Europe index, 3) the Morgan Stanley Far East index, 4) Frank Russell 2000, 5) Merrill Lynch T-Bill index, and 6) Merrill Lynch 10+ yr T-Bond index. We plot the geometric means and the annualized volatilities of the monthly returns over the twenty-three-year period in Exhibit 2.

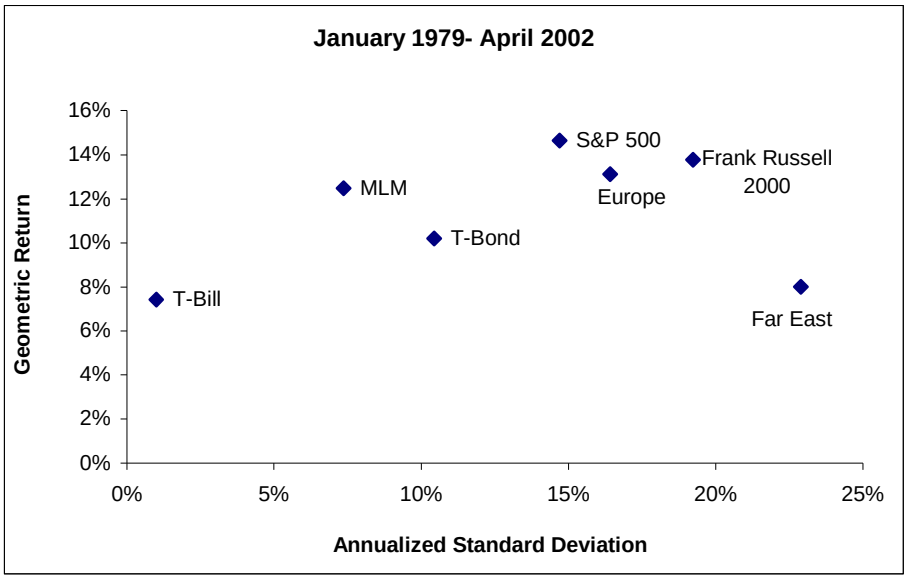

Exhibit 2: Return characteristics of the seven asset classes over January 1979 to April 2002. The MLM index is characterized by relatively high average geometric return and relatively low volatility.

To conduct a more detailed analysis, we broke up the data into three sections. The first was January 1979-August 1991, when interest rates and inflation were generally high and economic growth was relatively slow. The second was from September 1991 to March 2000, which was generally characterized by lower rates, reduced inflation, high growth and strong

stock-market performance. The last period from April 2000 to April 2002 had relatively poor stock-market performance, anemic economic growth and low rates. Plots of these periods are in Appendix I. This analysis gives evidence of the use of the MLM as a diversifier. A similar time-sectional breakdown could be done by choosing May 1989, the official launch date of the index, as the first breakpoint; since the returns prior to that date are hypothetical (Schneeweis and Spurgin [1996]).

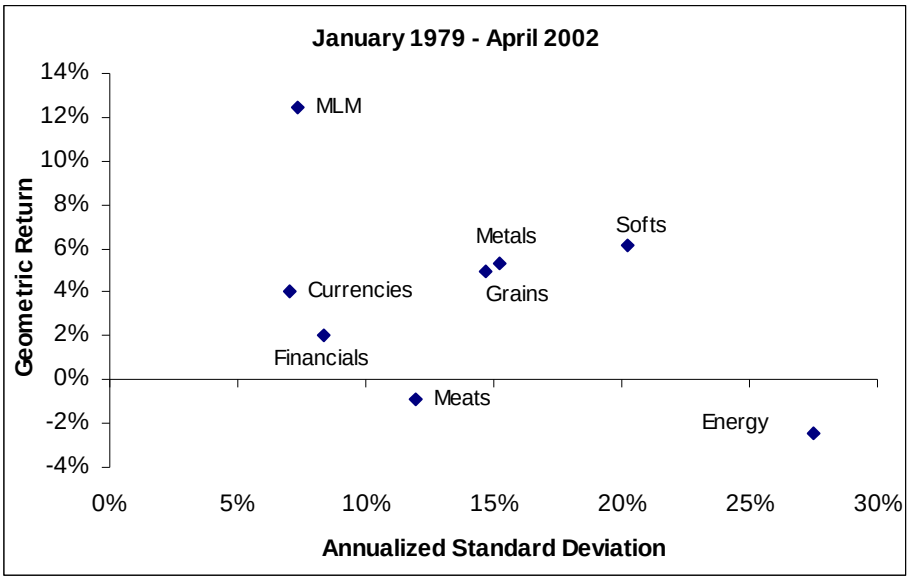

Exhibit 3: The returns of the Mt. Lucas sub-indices.
In general, the returns are volatile and can vary significantly depending on the period under observation. However, the MLM index itself is relatively stable. This is due to active rebalancing of the components and the diversity of asset returns.

Exhibit 3 provides evidence that the individual commodity classes generally underperformed the MLM index by a wide margin. However, the overall performance of the MLM index gave a return of just over 12% and a volatility around 7%. Sub-sector returns also vary considerably across the different time periods. However, the MLM index was more stable, its average return varying from about 16% during the 1980s to around 7% from April 2000 – April 2002. Plots of sector performance in different time periods are in Appendix I.

Exhibit 4 decomposes the MLM return into three distinct components. The first component equals the return from the T-Bills that are held in the margin account. Historically, T-

Bills have been the largest component of the MLM return; the decline in T-Bill returns and the lower volatility of individual commodities have been largely responsible for the decline in MLM returns over time. The second component equals the static return from the trend-following strategy. This has ranged from 3.5% in the 1980s to 1.75% in the most recent period. The third and critical component is the gains from the regular rebalancing of the MLM index. In the 1980s, when volatility was higher, the gains from rebalancing were approximately 3%. As volatility decreased during the 1990s and early 2000s, this gain reduced to approximately 0.5%. An economic explanation of these rebalancing gains is presented in Jaeger et al. [2002]. They claim that the rebalancing gains will become even more obvious when mean reversion of the components of the index is assumed, and that the mean reversion can be legitimately claimed to be present in interest rate movements and to some degree in commodity prices. This gives us confidence that these gains will occur in the future. There are other futures indices, such as the Dow Jones-AIG Commodity Index, that have been constructed to benefit from rebalancing gains.

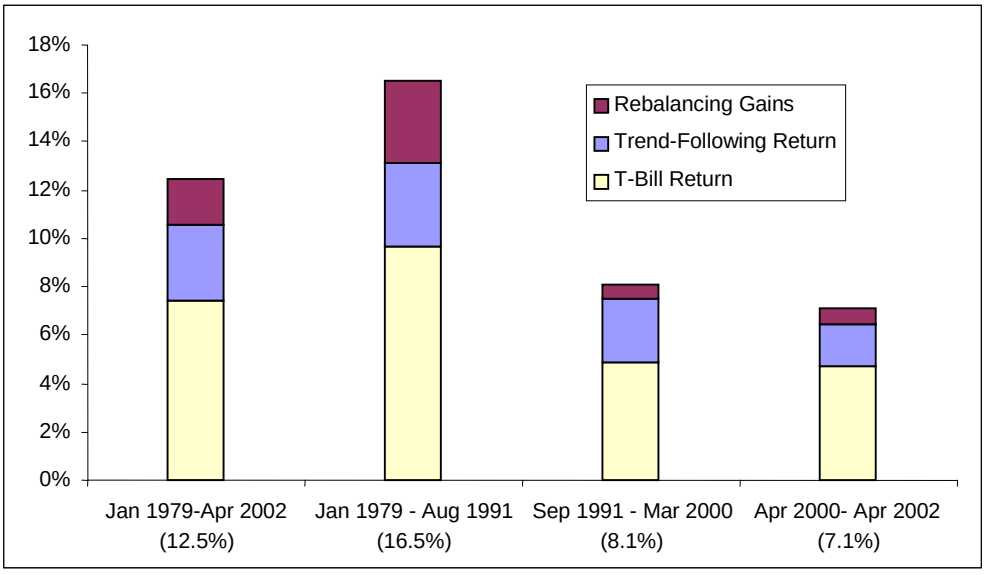

Exhibit 4: The return components of the MLM index.
The first component is from T-Bills that are held as margin. The second block is from the trend-following strategy and the last factor is the gains from the monthly rebalancing. The rebalancing gains are much higher in periods of increased volatility. Below each period, we report the volatility of the MLM index in that time-period.

Exhibit 5 shows the growth of four asset classes in the time period under review. The S&P 500 has had the highest growth to date, although accompanied with relatively large volatility. The MLM index's growth is just below that of the S&P500, with a smoother path. T-Bills display minimum volatility, whereas 10-Yr Bonds provide an intermediate path between equity and cash.

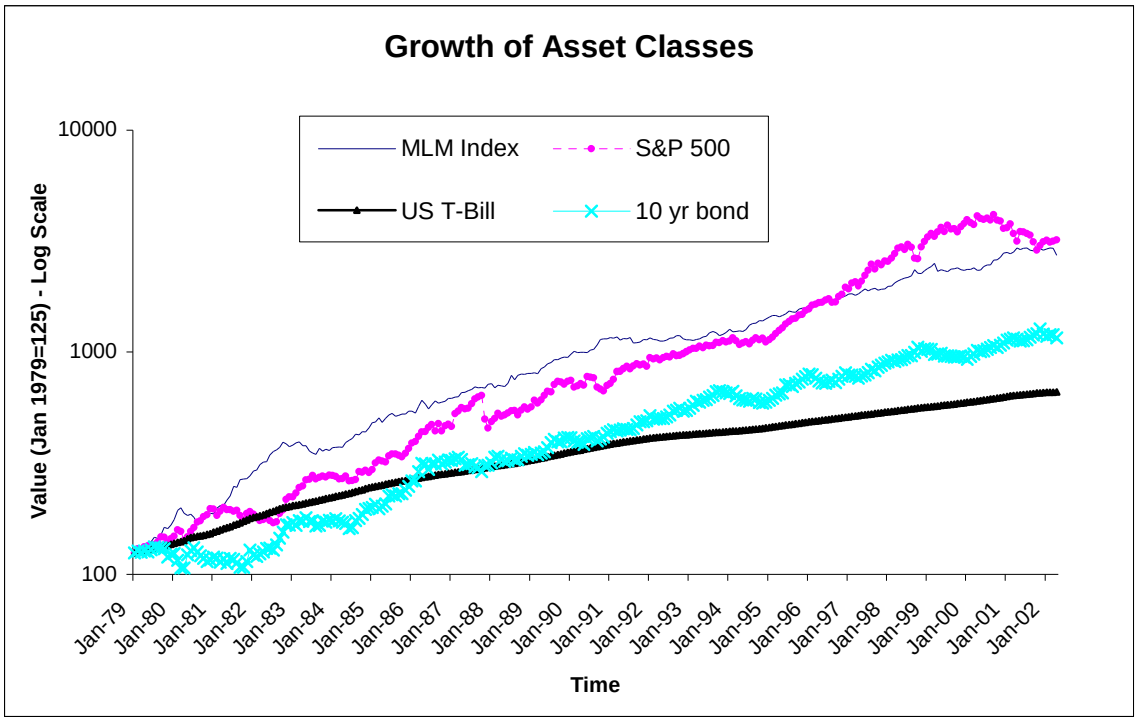

Exhibit 5: A comparison of the growth of the asset classes in the study from Jan 1979 to April 2002. The picture shows the how much $100 invested in January 1979 would be worth over time. The MLM index has slightly lower average growth but volatility is much less than the S&P 500.

Exhibit 6 tabulates the monthly correlations between the seven asset classes. As expected, the equity categories have large positive correlations with each other. The MLM index has slight positive correlation with T-Bills and T-Bonds, but is relatively uncorrelated with the equity classes. This is desirable as it suggests the index is a good diversifier as part of a portfolio. Exhibit 7 reports the correlations of the MLM components with the asset classes and amongst each other. None of the sectors are much correlated with each other, or with the equity and bond classes. Three of the seven sectors: metals, softs, and energy are weakly correlated with the

MLM index. This indicates the absence of a linear relationship between the MLM index and the traditional bond and equity classes suggesting useful portfolio benefits.

Asset Class	T-Bills	S&P 500	MLM	F. Russell 2000	Far East	T-Bond	Europe
T-Bills	1.00						
S&P 500	-0.02	1.00					
MLM	0.31	-0.05	1.00				
F. Russell 2000	-0.07	**0.78**	-0.10	1.00			
Far East	0.01	0.35	-0.01	0.29	1.00		
T-Bond	0.23	0.27	0.06	0.13	0.06	1.00	
Europe	-0.04	**0.61**	-0.04	**0.53**	**0.54**	0.21	1.00

Exhibit 6: Correlations between the seven asset classes
Equities are generally correlated with each other. The MLM has relatively low correlation with equities and is not highly correlated with any asset category. Correlations over 0.5 are in bold type.

Asset Class	T-Bills	S&P 500	MLM	F. Russell 2000	Far East	T-Bond	Europe	Currencies	Energy	Financials	Grains	Meats	Metals	Softs
Currencies	0.13	-0.08	0.32	-0.10	0.02	0.06	-0.05	1.00						
Energy	-0.03	0.00	**0.51**	-0.01	-0.04	-0.08	0.03	-0.02	1.00					
Financials	0.03	-0.10	0.16	-0.09	-0.04	0.00	-0.11	0.08	-0.01	1.00				
Grains	0.11	-0.02	**0.56**	-0.03	0.06	-0.05	0.00	0.07	0.01	0.04	1.00			
Meats	-0.06	0.05	-0.07	0.03	0.06	0.10	0.05	-0.08	-0.10	-0.05	-0.08	1.00		
Metals	0.22	-0.07	0.44	-0.07	0.12	0.11	0.00	0.23	0.05	-0.15	0.02	-0.01	1.00	
Softs	0.12	0.02	**0.52**	-0.06	-0.11	0.12	-0.04	-0.02	0.00	0.19	0.17	-0.07	0.09	1

Exhibit 7: Correlation coefficients for the MLM components.
The MLM index is weakly correlated with three of its components. The sector indices are generally uncorrelated with each other and with the other asset classes. Correlations over 0.5 are in bold type.

In Exhibit 8, we analyze the non-linear nature of the relationship between the S&P500 and the MLM index. We sort the S&P500 observations and place them into quintiles (56 months each). From the graph, we see that the MLM index has positive average returns in all five states of S&P500 performance. Its best performance occurs when the S&P500 has the worst performance, suggesting that the MLM asset class can be a source of positive returns *and* diversification.

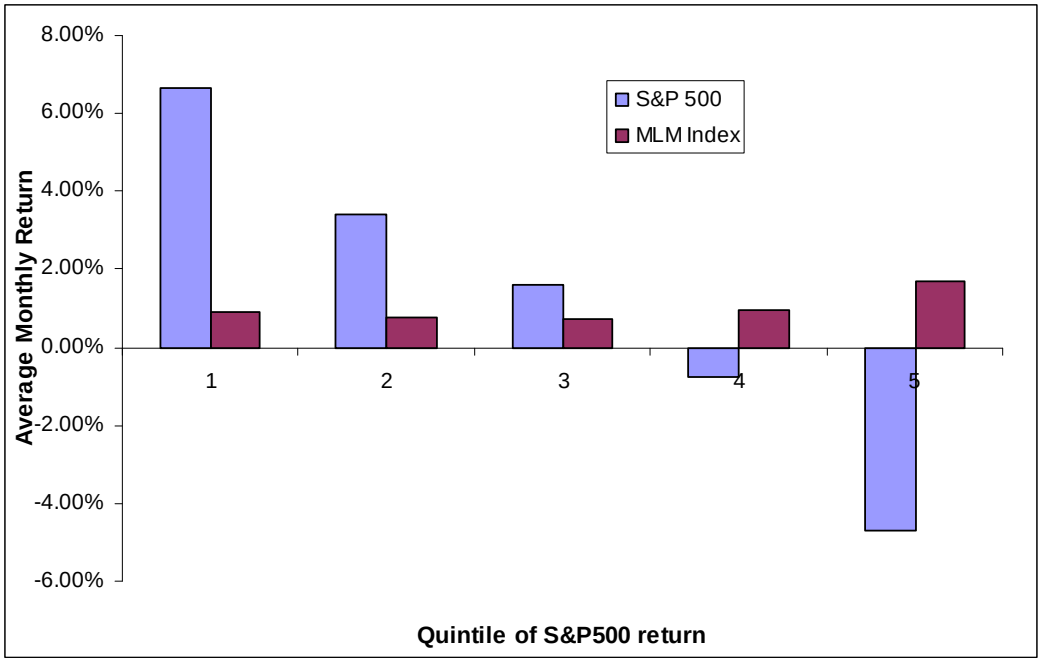

Exhibit 8: Relative performance of the MLM index in different states of the world. The S&P500 months are broken into five equally sized groups ranging from the best months (group 1) to the worst (group 5). The average return of these groups is plotted along with the average return of the MLM index. The MLM index seems to perform independently of the S&P500, but has its highest payoff when the S&P500 does poorly.

Next, we evaluate the assets via the CAPM model (with market = S&P500). In Exhibit 9, as expected, most equity categories display a systematic risk-return relationship with the S&P500, as indicated by the clustering of these points around the market line. In contrast, and significantly, the MLM index itself has a slightly negative beta, but the returns are much higher than expected, given this CAPM model. This indicates that the S&P500 may not explain much of the MLM index's return, indicating its use a diversifier.

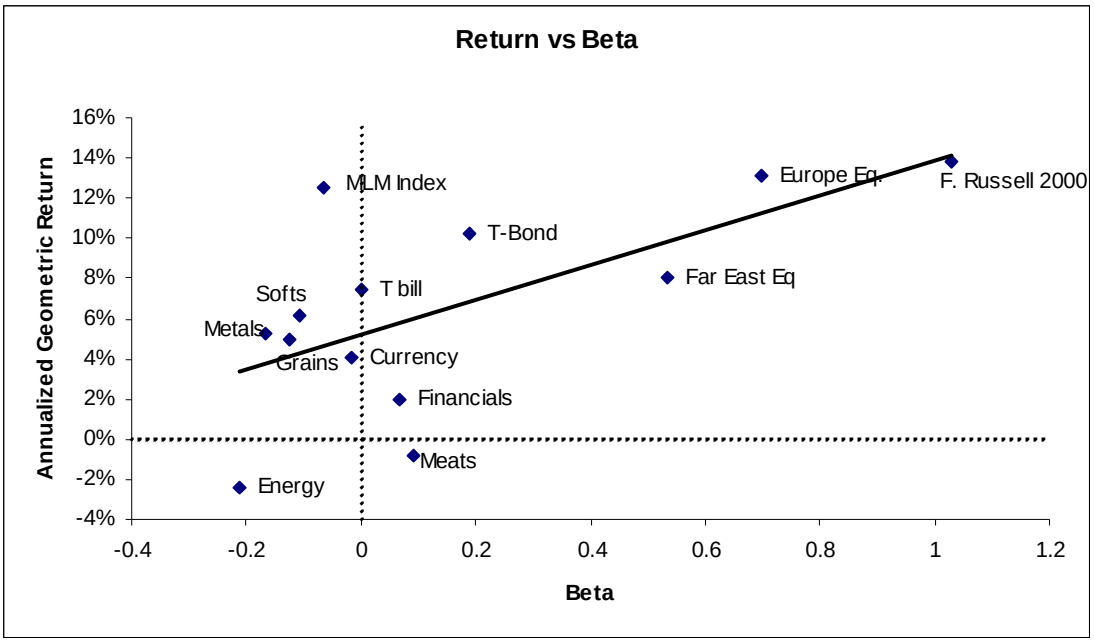

Exhibit 9: Plots of asset return as a function of market risk.
The relationship is generally systematic for the traditional asset classes in equity and bonds. With the commodity classes, there is little relationship between beta and return. The two MLM asset classes seem to have low market risk and yet a relative high return, as evaluated by the CAPM model.

THE MULTI-PERIOD PORTFOLIO MODEL

Single-period mean-variance (MV) analysis has gained widespread popularity; practitioners have successfully applied it to areas such as asset allocation, mutual fund construction, and financial planning. In such models, decisions are typically made at the beginning of the planning horizon. For long-term investors, including pension plans, university endowments and individuals, this approach has several shortcomings. These investors have liabilities and goals that must be met at future dates and such temporal issues are difficult to address using a static single-period framework. Also, as we will see, a multi-period portfolio model can take advantage of volatility by rebalancing the asset mix in order to increase returns.

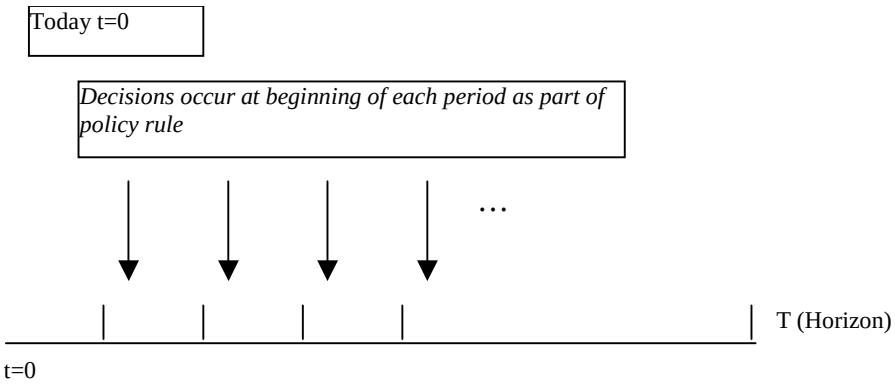

Exhibit 10: A representation of a multi-period model.
The planning horizon is divided into T periods and a decision regarding the asset allocation is made at the beginning of each period using a policy rule. One can then use optimization to find a good solution to the problem.

Exhibit 10 illustrates how one might formulate a multi-period model. We discretize the planning period t={1…T} and make decisions via a policy rule at the beginning of each period. One such rule is fixed-mix in which allocations are rebalanced in each period to fixed target proportions; the fixed-mix rule is optimal for many long-term investors (Samuelson [1969]). Another possibility is to solve a sequence of single period MV models, one at each decision period. Policy rules should be tailored to suit the particular characteristics and goals of an investor.

One needs to be careful in choosing a risk/reward performance criterion for multi-period models. For example, volatility in asset returns can be harnessed over time to boost portfolio performance, a feature studied by Mulvey et al. [2001]. By regularly rebalancing allocations between volatile assets, one can obtain additional return over a naïve buy-and-hold strategy. Exhibit 11 shows how the rebalancing inherent in the fixed-mix policy rule improves on the single-period efficient frontier. Luenberger [2000] calls this effect “volatility pumping” (herein, rebalancing gains). The effect is most pronounced when funds are rebalanced between assets that have low correlation with each other, positive expected returns, and high individual volatility. In this context, popular measures such as the Sharpe ratio may be misleading in multi-period models, as they do not recognize rebalancing gains.

As a further advantage, the multi-period setting can address transaction and impact costs in a direct fashion. For example, investors may wish to consider the impact of taxes when selling an asset. Unfortunately, it is difficult to analyze such issues in a single period model. See, for example, Mulvey and Simsek [2002].

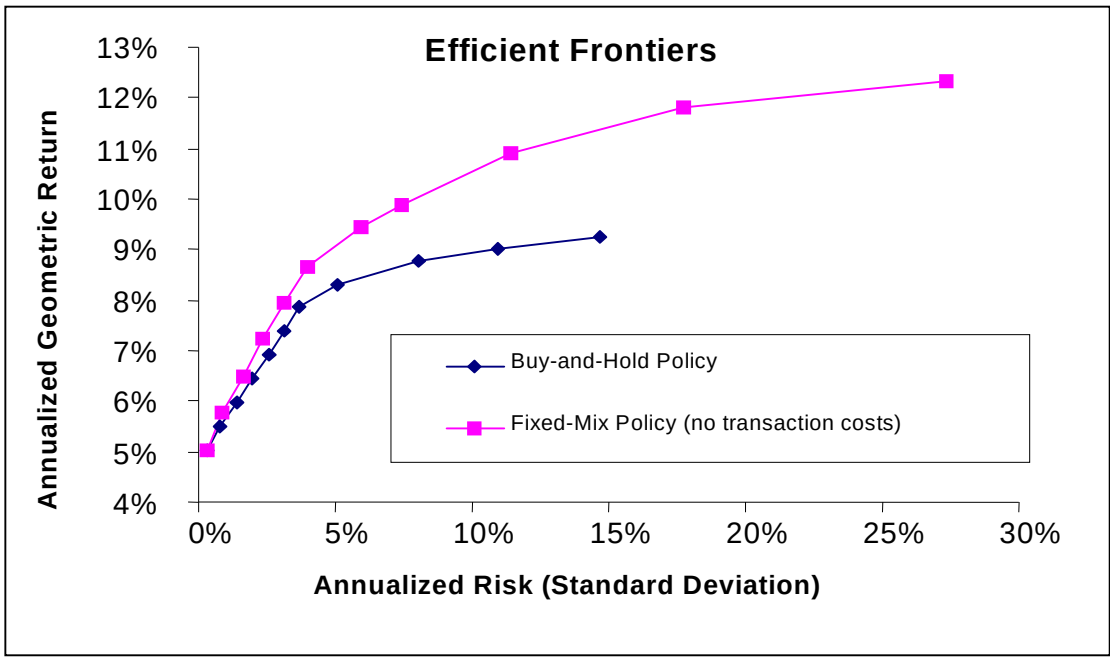

Exhibit 11: A comparison of single-period and multi-period efficient frontiers. The multi-period model employs the fixed-mix policy rule in which allocations are rebalanced each quarter to a fixed target proportion. Regular rebalancing leads to a "buy low, sell high" strategy that over time leads to higher growth and lower volatility. (Figure adapted from Mulvey et al. [2003])

To illustrate the advantages of rebalancing a portfolio, we use historical data from 1972 to 1997. The opportunity set consists of four asset classes, one of which is a commodity index comparable to the MLM index: S&P 500 (large US companies), EAFE (foreign stocks from Europe, Australia, and Far East), REIT (real estate investment trusts), and GSCI (Goldman Sachs commodity index).

The historical returns are plotted in risk/return space in Exhibit 12, indicating compound returns and risk via standard deviation of return over the 26-year period. We show that the return/risk characteristics of dynamically balanced portfolios dominate any individual asset category. In fact, this type of relationship is common over extended periods, for a wide variety of

markets and alternative time periods. Multi-asset portfolio performance will be even better when individual asset categories possess greater volatility and are relatively uncorrelated with each other.

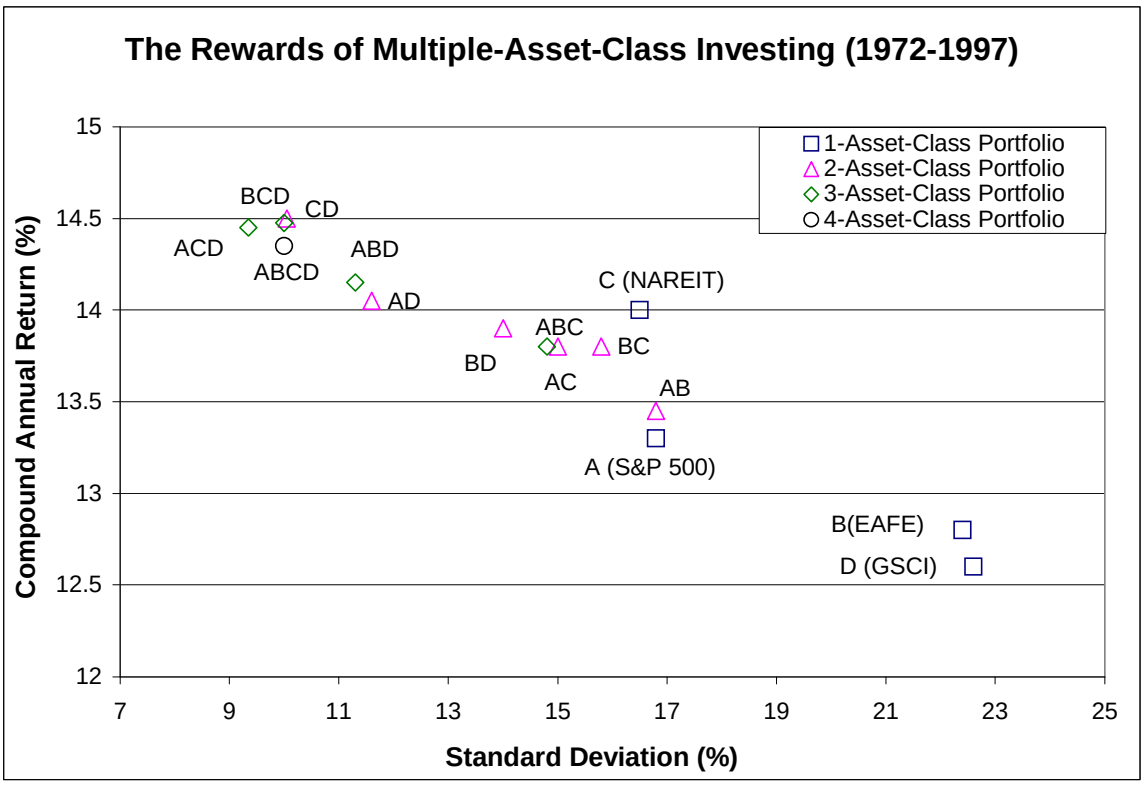

Exhibit 12: Reward/risk characteristics for several fixed-mix strategies. Dynamically rebalanced portfolios dominate individual asset categories. (Figure adapted from Mulvey and Simsek [2002])

EMPIRICAL STUDY

As mentioned earlier, we employ the fixed-mix policy rule to analyze the effects of adding the MLM index into the opportunity set. Also, to observe the gains due to rebalancing within the portfolio, we solve the problem under the buy-and-hold policy assumptions for the asset universe that involves the index. For simplicity, we ignore factors such as transaction costs and taxes, though these can be included. See Mulvey and Simsek [2002] for a discussion of modeling transaction costs. We optimize the risk-adjusted geometric return of the portfolio. For the risk measure, we use the annualized standard deviation of monthly returns. This captures the

volatility experienced by a long-term investor across the planning period. The mathematical representation of the model is in appendix II. For purposes of this analysis, we leverage the MLM index by a factor of 1.3, a relatively conservative factor.

RESULTS

Exhibit 13 shows the efficient frontiers resulting from the optimization models described above. The figure indicates that there is an advantage to be gained by investing in the basic MLM asset class for multi-period investors, regardless of the policy rule. If the mix is rebalanced within the portfolio even further gains are realized. The lowest efficient frontier results from excluding the MLM asset class entirely. Over this period, the highest return one can hope to gain is about 15% with roughly 14% volatility. By including the MLM index under a fixed-mix policy, one was able to get 15% returns with 8% volatility. Risk-tolerant investors can also gain greater absolute returns with considerably less risks. Of course, increasing the leverage factor in the MLM index (to 2 or 3) will expand the efficient frontier upwards and the proportion of the MLM asset class in the portfolio will increase. Herein, a monthly rebalancing scheme is pursued. Lower rebalancing frequencies (3 to 6 months) may lead to superior portfolio performance, since they are associated with negatively auto-correlated returns for the trend-following components (Rulle [2003]).

One of the most important problems regarding the evaluation of alternative investments (hedge funds, in our case) is that they may not be readily rebalanced at each period, due to problems like survivorship bias and lockup periods. Krishnan and Nelken [2003] discuss how an investor should be compensated and what illiquidity premium they should receive when they can't rebalance their investments. They develop a technique for calculating the illiquidity haircut for a hedge fund and claim that lower returns should be acceptable for more liquid investments (for example, interest rate futures) and especially for ones that provide rebalancing gains to a portfolio.

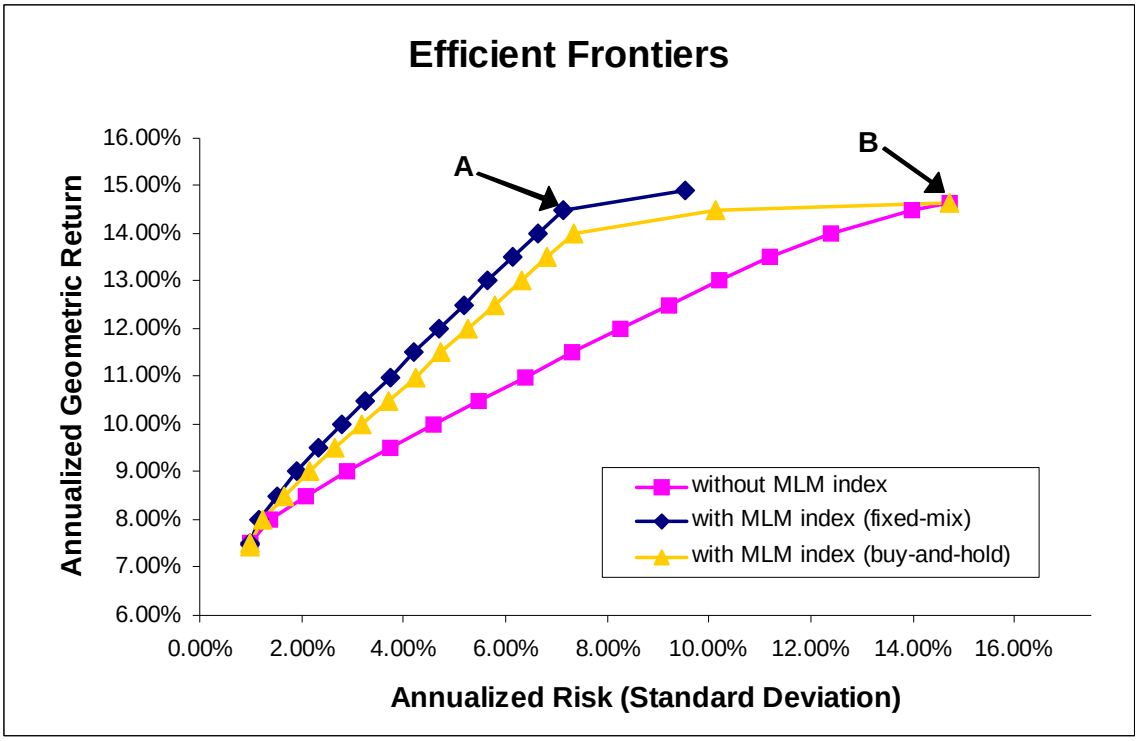

Exhibit 13: The efficient frontiers with and without the MLM asset class. The lower efficient frontier is obtained when MLM is not included. The middle one is achieved when the index is included but portfolio rebalancing is not allowed. The upper frontier shows that one can achieve similar returns with much lower risk by using the MLM index under the fixed-mix policy rule. The growth of points labeled A and B is compared in Exhibit 14.

A time-series representation of values of portfolios A and B (labeled in Exhibit 13) is given in Exhibit 14. The graph shows what the value of $100 invested in 1980 in each portfolio would be today. We can see the final values for both portfolios is almost the same, but portfolio A has a smoother growth path than portfolio B. Appendix III lists the composition of the respective efficient portfolios.

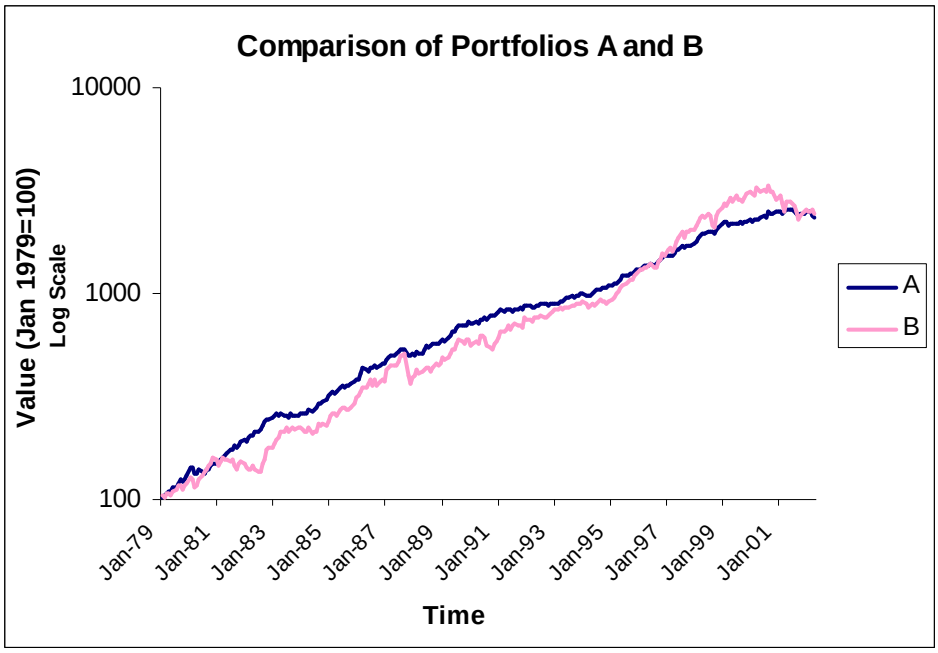

Exhibit 14: The evolution of various portfolios over time.
The path for portfolio B is more volatile than the path for portfolio A, but the average growth rate is the same. See Exhibit 13 for the location of each point on the efficient frontier.

PROPOSED MODIFICATIONS TO MLM INDEX

Generally, margin accounts are held in T-Bills, so that the investor earns the T-Bill return on the investment in addition to the return on the commodity trading. For long-term investors, there is risk involved in rolling over short-term T-Bill instruments because there is uncertainty in the rate at which returns will be reinvested (Campbell and Viceira [2002]. Thus, the ideal risk-free asset for long-term investors would be an inflation-indexed long-term bond as this locks-in real interest rates for the duration of the investment. In periods of low inflation, a good alternative is a long-term treasury bond.

Long-term bonds have an important advantage since interest rates tend to fall in periods when economic activity is bleak, and hence, traditional assets perform poorly. This causes a drop in long-term bond yields leading to a rise in prices. We generate historical returns for this asset by replacing T-Bills with T-Bonds in the MLM index.

We add this new asset class and rerun our optimization model to get a third efficient frontier shown in Exhibit 15. We see that there is an improvement at the top-end of the efficient frontier. Risk-tolerant investors could have obtained around 18% return with roughly 13% volatility. By comparison, the S&P 500 has 15% return and 15% volatility.

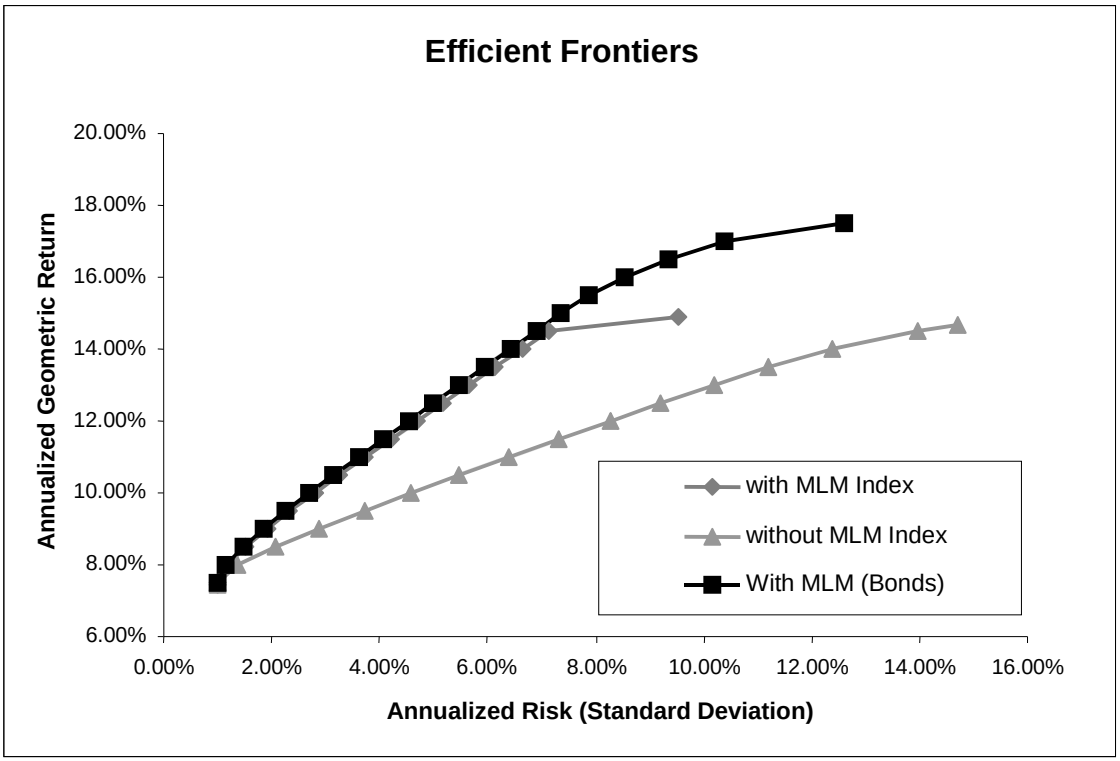

Exhibit 15: Adding MLM (Bonds) to the analysis.
The top end of the frontier expands showing increased investment possibilities when one adds MLM (Bonds) to the mix.

CONCLUSIONS

Our analysis provides further evidence that the MLM index is a useful investment, since the asset category has low correlation with the market and other traditional asset classes, making it a good diversifier, and generally gives a positive payoff when traditional assets perform badly. These properties, when combined with its volatility make it useful in a multi-period asset allocation model. Investors may obtain superior performance when they diversify portfolios to include the MLM index with traditional asset categories such as the S&P500. We highlighted the

differences between single-period and multi-period asset allocation models. In particular, long-term investors can harness volatility using multi-period models, and so volatility can be desirable when it is employed properly. Commodity futures investments are easily leveraged and the resulting volatility can improve portfolio performance. We also propose replacing T-Bills held in the margin account by T-Bonds, arguing that this enhances the index's role in multi-period asset allocation.

The fundamental concepts for the MLM index are regular rebalancing and trend-following. The argument that the trend-following strategies "buy" volatility is more complicated than simply following a market trend. Rebalancing gains play a significant role in enhancing the long-term returns, especially during periods of high volatility. In this regard, combining trend-following and rebalancing generates the superior performance of this distinct asset class. Further research could evaluate other investment strategies as well as other classes of alternative investments to look for similar patterns.

ENDNOTES

Kaul was a MSE student in the ORFE Department at Princeton University during the time of this writing.

ACKNOWLEDGEMENT

We are grateful to an anonymous reviewer for recommendations on improving the paper.

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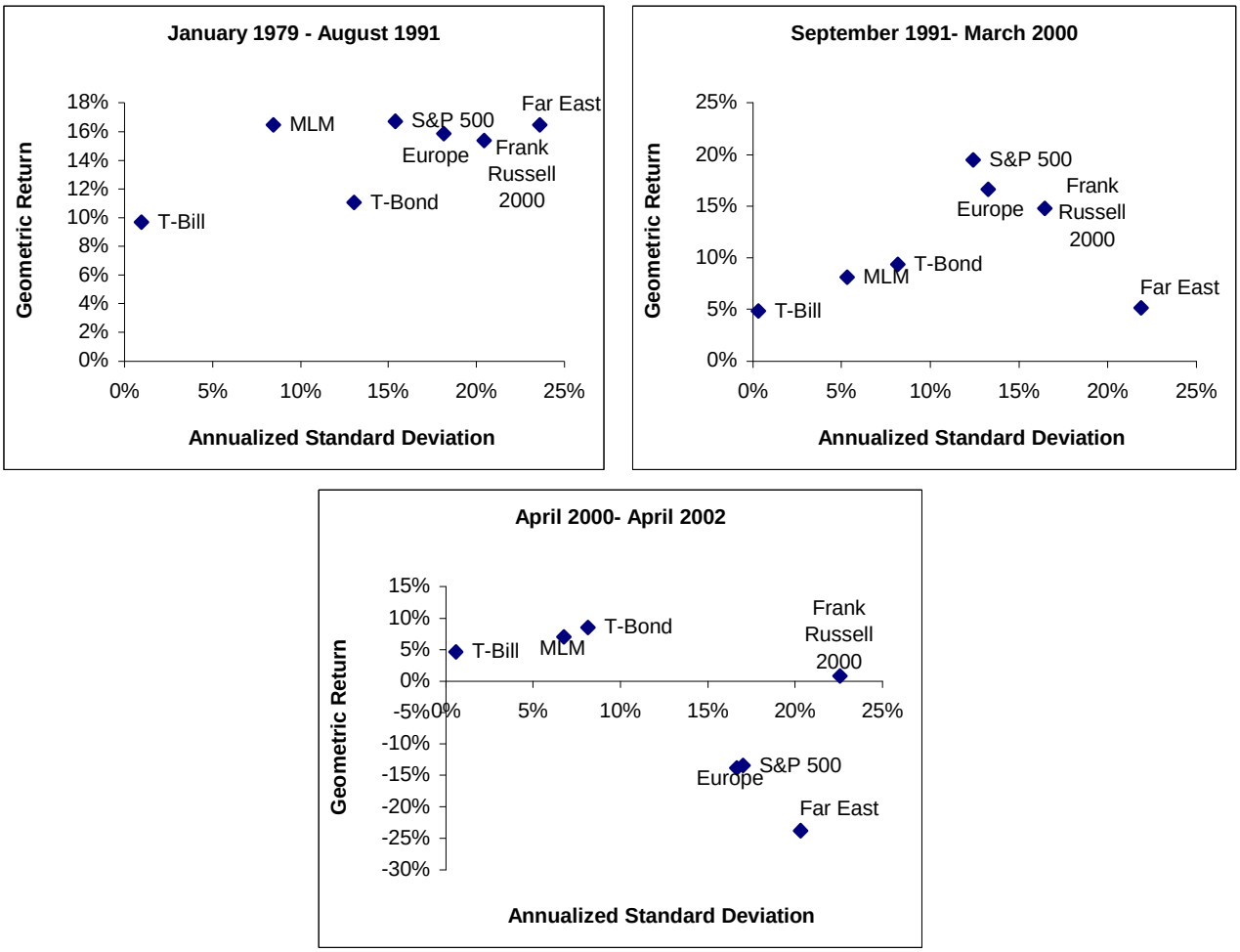
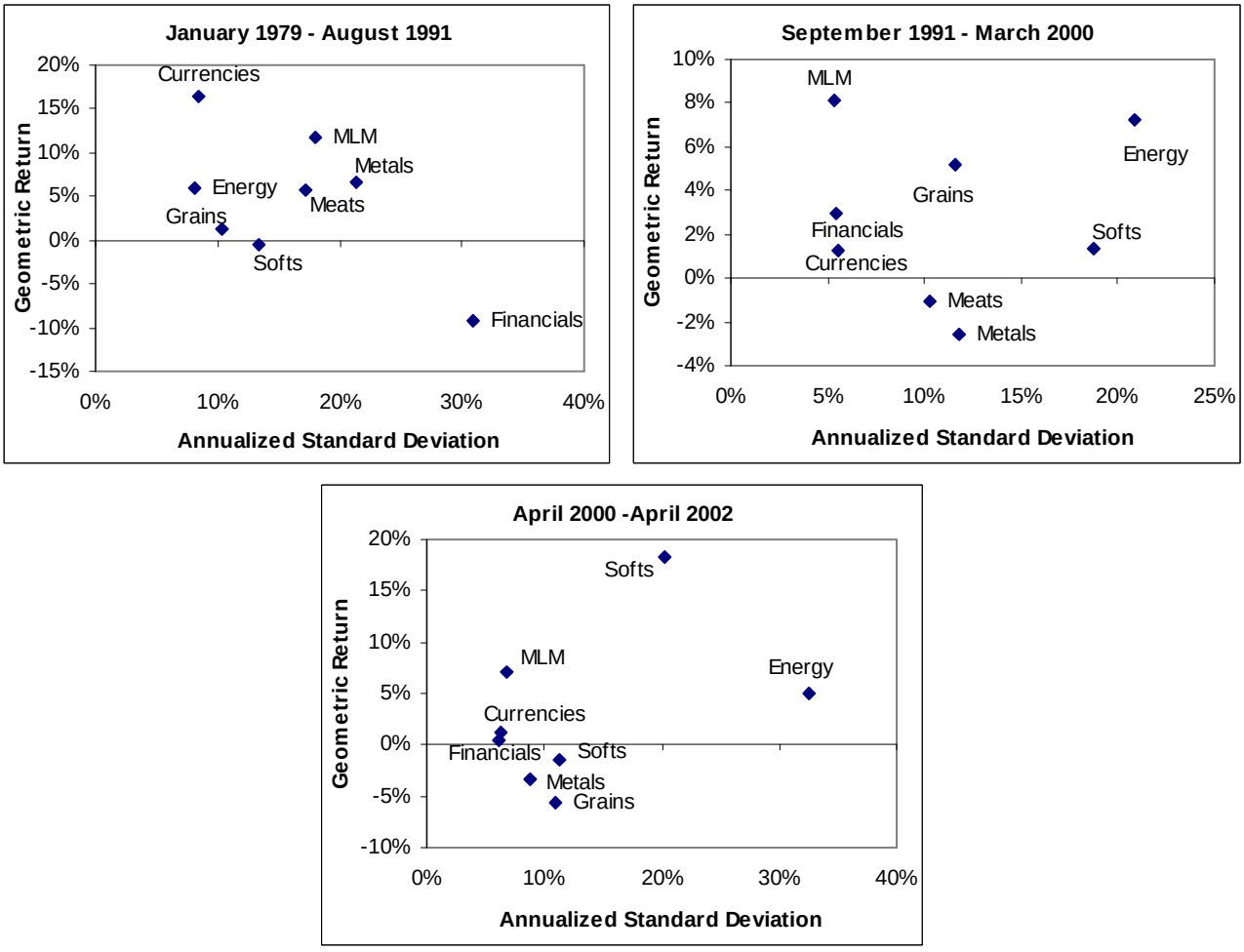
APPENDIX I



Risk-return plots of the seven asset classes over the time periods Jan. 1979 – Aug. 1991, Sep. 1991 – Mar 2000, and Apr 2000 – Apr. 2002.



Risk-return plots of the MLM asset class and its constituent sectors over the time periods Jan. 1979 – Aug. 1991, Sep. 1991 – Mar 2000, and Apr 2000 – Apr. 2002.

APPENDIX II

A mathematical program for a multi-period portfolio model using the fixed-mix rule

$$\max_{x_i} \alpha \left(\left[\prod_{t=1}^{T} \sum_{i=1}^{N} x_i (1 + r_{i,t}) \right]^{12/T} - 1 \right) - (1 - \alpha) \sqrt{12 \sum_{t=1}^{T} \frac{(R_t - E[R])^2}{T - 1}} \quad (1)$$

Subject To

$$R_t = \sum_{i=1}^{N} x_i (1 + r_{i,t}) \quad (2)$$

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0 \quad (3)$$

The objective function (1) rewards a higher return and penalizes greater risk. It weights the final wealth after T periods by α, and risk by $(1-\alpha)$. For risk, we use the standard deviation of monthly returns, as it conveys the expected period-to-period volatility of the investment strategy being evaluated. Equation (2) defines the portfolio return in each period, R_t. By ignoring transaction costs, we are able to eliminate the need for keeping track of allocations before and after rebalancing. Equation (3) simply states that the portfolio weights must add up to one and that short selling is not allowed. By solving this optimization problem for different values of α, we trace out the efficient frontier.

APPENDIX III

Efficient Portfolios obtained from optimization

Point	S&P 500	3 MONTH T-BILL	FRANK RUSSELL 2000	FAR EAST EQUITY	TREASURY 10+ YEARS	EUROPE EQUITY	Risk	Return
1	0.00%	99.52%	0.28%	0.00%	0.00%	0.19%	0.99%	7.47%
2	0.17%	99.06%	0.39%	0.00%	0.00%	0.37%	1.00%	7.50%
3	5.17%	92.08%	0.12%	0.00%	1.12%	1.51%	1.38%	8.00%
4	9.66%	84.30%	0.01%	0.00%	3.51%	2.51%	2.08%	8.50%
5	14.11%	76.43%	0.00%	0.00%	5.95%	3.51%	2.88%	9.00%
6	18.61%	68.45%	0.00%	0.00%	8.42%	4.52%	3.72%	9.50%
7	23.17%	60.36%	0.00%	0.00%	10.94%	5.54%	4.59%	10.00%
8	27.79%	52.16%	0.00%	0.00%	13.49%	6.57%	5.49%	10.50%
9	32.47%	43.84%	0.00%	0.00%	16.08%	7.61%	6.39%	11.00%
10	37.21%	35.38%	0.00%	0.00%	18.72%	8.68%	7.32%	11.50%
11	42.03%	26.79%	0.00%	0.00%	21.42%	9.75%	8.26%	12.00%
12	46.93%	18.06%	0.00%	0.00%	24.18%	10.83%	9.22%	12.50%
13	51.92%	9.16%	0.00%	0.00%	26.99%	11.93%	10.20%	13.00%
14	56.98%	0.10%	0.00%	0.00%	29.88%	13.04%	11.19%	13.50%
15	70.02%	0.00%	0.00%	0.00%	16.89%	13.09%	12.38%	14.00%
16	84.80%	0.00%	0.00%	0.00%	2.19%	13.01%	13.96%	14.50%
17	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14.70%	14.65%

Table 1: Efficient portfolios when MLM is excluded

Point	S&P 500	3 MONTH T-BILL	FRANK RUSSELL 2000	FAR EAST EQUITY	TREASURY 10+ YEARS	EUROPE EQUITY	MLM INDEX	Risk	Return
1	0.00%	99.81%	0.16%	0.02%	0.00%	0.01%	0.00%	0.99%	7.45%
2	0.06%	99.46%	0.20%	0.00%	0.00%	0.06%	0.21%	1.00%	7.50%
3	0.76%	95.07%	0.38%	0.00%	0.99%	0.12%	2.68%	1.24%	8.00%
4	1.56%	90.00%	0.56%	0.00%	2.35%	0.20%	5.33%	1.66%	8.50%
5	2.51%	84.58%	0.73%	0.00%	3.61%	0.28%	8.28%	2.15%	9.00%
6	3.64%	78.78%	0.90%	0.00%	4.76%	0.37%	11.55%	2.66%	9.50%
7	4.98%	72.55%	1.06%	0.00%	5.77%	0.46%	15.17%	3.18%	10.00%
8	6.56%	65.87%	1.23%	0.00%	6.64%	0.55%	19.15%	3.70%	10.50%
9	8.40%	58.67%	1.39%	0.00%	7.37%	0.63%	23.54%	4.22%	11.00%
10	10.55%	50.92%	1.56%	0.00%	7.92%	0.69%	28.36%	4.74%	11.50%
11	13.05%	42.55%	1.73%	0.00%	8.29%	0.72%	33.66%	5.27%	12.00%
12	15.94%	33.52%	1.90%	0.00%	8.47%	0.70%	39.48%	5.79%	12.50%
13	19.27%	23.75%	2.08%	0.00%	8.43%	0.62%	45.85%	6.31%	13.00%
14	23.10%	13.17%	2.27%	0.00%	8.16%	0.47%	52.83%	6.83%	13.50%
15	27.49%	1.71%	2.47%	0.00%	7.64%	0.21%	60.48%	7.35%	14.00%
16	69.81%	0.00%	0.00%	0.00%	0.00%	0.00%	30.19%	10.11%	14.50%
17	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14.70%	14.65%

Table 2: Efficient portfolios when MLM index is included (under buy-and-hold policy)

Point	S&P 500	3 MONTH T-BILL	FRANK RUSSELL 2000	FAR EAST EQUITY	TREASURY 10+ YEARS	EUROPE EQUITY	MLM INDEX	Risk	Return
1	0.00%	99.52%	0.28%	0.00%	0.00%	0.19%	0.00%	0.99%	7.47%
2	0.17%	99.06%	0.39%	0.00%	0.00%	0.37%	0.00%	1.00%	7.50%
3	2.22%	92.24%	0.36%	0.00%	0.00%	0.77%	4.41%	1.18%	8.00%
4	4.17%	85.30%	0.35%	0.00%	0.19%	1.15%	8.84%	1.50%	8.50%
5	5.97%	78.06%	0.39%	0.00%	0.94%	1.47%	13.17%	1.90%	9.00%
6	7.74%	70.81%	0.46%	0.00%	1.67%	1.79%	17.52%	2.34%	9.50%
7	9.54%	63.55%	0.51%	0.00%	2.42%	2.11%	21.87%	2.79%	10.00%
8	11.34%	56.29%	0.56%	0.00%	3.15%	2.44%	26.23%	3.26%	10.50%
9	13.12%	49.01%	0.61%	0.00%	3.89%	2.78%	30.59%	3.73%	11.00%
10	14.91%	41.72%	0.68%	0.00%	4.65%	3.09%	34.94%	4.21%	11.50%
11	16.72%	34.43%	0.72%	0.00%	5.40%	3.42%	39.31%	4.69%	12.00%
12	18.49%	27.13%	0.78%	0.00%	6.13%	3.76%	43.71%	5.17%	12.50%
13	20.30%	19.80%	0.84%	0.00%	6.92%	4.06%	48.07%	5.66%	13.00%
14	22.11%	12.47%	0.89%	0.00%	7.68%	4.39%	52.46%	6.15%	13.50%
15	23.91%	5.14%	0.94%	0.00%	8.44%	4.71%	56.86%	6.64%	14.00%
16	27.61%	0.00%	0.64%	0.00%	5.38%	4.83%	61.54%	7.14%	14.50%
17	63.09%	0.00%	0.43%	0.00%	0.00%	0.00%	36.48%	9.53%	14.92%

Table 3: Efficient portfolios when MLM index is included (under fixed-mix policy)

Point	S&P 500	3 MONTH T-BILL	FRANK RUSSELL 2000	FAR EAST EQUITY	TREASURY 10+ YEARS	EUROPE EQUITY	MLM INDEX (Bonds)	MLM INDEX	Risk	Return
1	0.00%	99.52%	0.28%	0.00%	0.00%	0.19%	0.00%	0.00%	0.99%	7.47%
2	0.06%	99.11%	0.39%	0.00%	0.00%	0.33%	0.11%	0.00%	1.00%	7.50%
3	2.19%	93.20%	0.18%	0.00%	0.00%	0.56%	1.76%	2.11%	1.16%	8.00%
4	4.18%	86.98%	0.04%	0.00%	0.00%	0.81%	2.89%	5.09%	1.47%	8.50%
5	6.02%	80.52%	0.00%	0.00%	0.40%	1.02%	3.98%	8.06%	1.85%	9.00%
6	7.77%	73.93%	0.00%	0.00%	0.99%	1.22%	5.04%	11.05%	2.27%	9.50%
7	9.52%	67.31%	0.00%	0.00%	1.62%	1.42%	6.08%	14.05%	2.71%	10.00%
8	11.29%	60.71%	0.00%	0.00%	2.22%	1.61%	7.13%	17.05%	3.16%	10.50%
9	13.05%	54.09%	0.00%	0.00%	2.83%	1.80%	8.17%	20.06%	3.61%	11.00%
10	14.76%	47.45%	0.00%	0.00%	3.46%	2.03%	9.22%	23.07%	4.07%	11.50%
11	16.52%	40.82%	0.00%	0.00%	4.07%	2.23%	10.27%	26.08%	4.54%	12.00%
12	18.27%	34.19%	0.00%	0.00%	4.69%	2.43%	11.33%	29.09%	5.01%	12.50%
13	20.03%	27.54%	0.00%	0.00%	5.31%	2.63%	12.38%	32.10%	5.48%	13.00%
14	21.79%	20.89%	0.00%	0.00%	5.93%	2.83%	13.44%	35.12%	5.95%	13.50%
15	23.55%	14.24%	0.00%	0.00%	6.54%	3.04%	14.49%	38.15%	6.42%	14.00%
16	25.32%	7.56%	0.00%	0.00%	7.17%	3.24%	15.53%	41.18%	6.90%	14.50%
17	27.08%	0.86%	0.00%	0.00%	7.83%	3.41%	16.59%	44.22%	7.37%	15.00%
18	30.50%	0.00%	0.00%	0.00%	2.95%	2.65%	22.55%	41.35%	7.89%	15.50%
19	34.01%	0.00%	0.00%	0.00%	0.00%	0.61%	32.09%	33.28%	8.52%	16.00%
20	35.44%	0.00%	0.00%	0.00%	0.00%	0.00%	45.51%	19.05%	9.35%	16.50%
21	36.45%	0.00%	0.00%	0.00%	0.00%	0.00%	59.77%	3.78%	10.40%	17.00%
22	15.53%	0.00%	0.00%	0.00%	0.00%	0.00%	84.47%	0.00%	12.61%	17.50%

Table 4: Efficient portfolio when both the MLM index and MLM (Bonds) are included