

**Hedge Fund Portfolios: Adding Value  
through Active Style Allocation Decisions**

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- Introduction
  - Selection of Hedge Fund Styles Based on Marginal Impacts on Portfolio Distribution
  - Enhanced Estimates for Parameters of Hedge Fund Return Distributions
  - Extending Black-Litterman Analysis to Active Style Allocation Decisions with Hedge Funds
  - Conclusion

# Introduction

## *Style Allocation Versus Fund Picking*

- Three types of value added by alternative multi-management industry
  - Strategic style allocation decisions
  - Tactical style allocation decisions
  - Fund picking decisions
- We have (strong) evidence that style allocation decisions matter (at least) as much as fund picking decisions

	Total Added Value	Value Added at Strategic Allocation Level	Value Added at Tactic Allocation Level	Value Added at Fund Picking Level
Average	0.58%	0.56%	-0.12%	0.14%
Standard Deviation	3.44%	4.50%	0.65%	4.18%
% Positive	55.06%	52.81%	43.82%	43.26%
Average Added Value	2.84%	3.50%	0.35%	3.43%
% Negative	44.94%	47.19%	56.18%	56.74%
Average Added Value	-2.18%	-2.72%	-0.48%	-2.34%

Based on 178 funds of hedge funds in the AAC database with data from January 1997 to December 2004

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# Advanced Techniques for Selection

## *Risk Measures*

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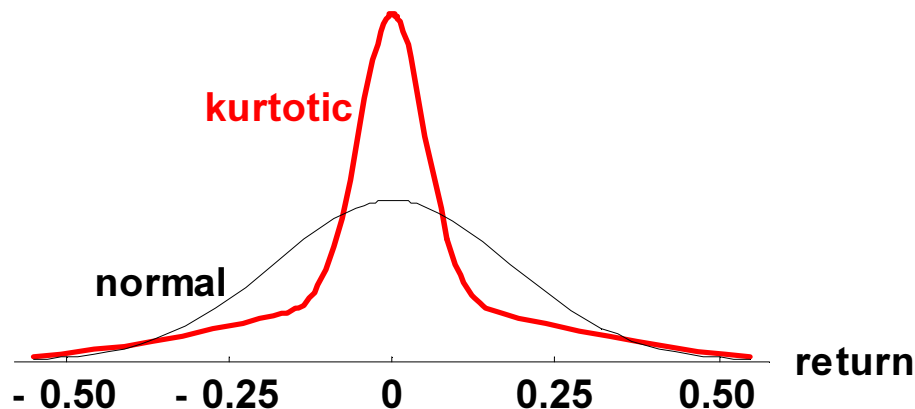
- Optimal asset allocation decisions can help dramatically improve the risk/return profile of the portfolio
- Hedge funds have appealing diversification properties with respect to stock and bond portfolios due to their alternative beta exposure
- Portfolio allocation exercises aim to find the best possible trade-off between risk and return, where risk can typically be measured in terms of
  - Volatility (measure of average risk, sufficient risk statistics for normally distributed returns)
  - VaR (measure of extreme risk, needed when returns are not normally distributed)

# Advanced Techniques for Selection

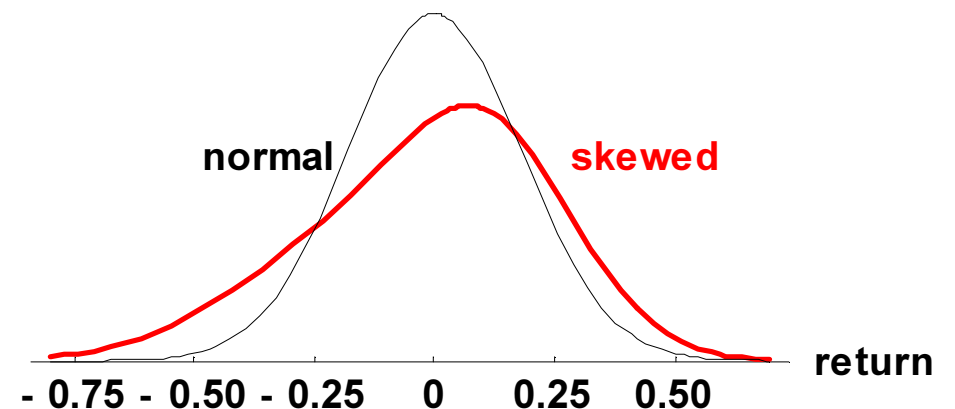
## *Skewness and Kurtosis*

- Asset returns are not normally distributed
  - The **kurtosis** (literally, "fat tails") of a distribution: measures frequency of large positive or negative asset returns
  - The **skewness** of a distribution: measures frequency with which large returns occur in a particular direction

Kurtotic and normal distributions



Skewed and normal distributions



# Advanced Techniques for Selection

## *Definitions of Co-Moments and Co-Cumulants*

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Higher order moments:

$$\begin{aligned}\mu^{(2)}(R) &= E\left[(R - E(R))^2\right] \\ \mu^{(3)}(R) &= E\left[(R - E(R))^3\right] \longrightarrow \text{Skewness after proper normalization} \\ \mu^{(4)}(R) &= E\left[(R - E(R))^4\right] \longrightarrow \text{Kurtosis after proper normalization}\end{aligned}$$

Higher order co-moments:

$$\begin{aligned}\text{CoV}(R_i, R_j) &= E\left[(R_i - E(R_i))(R_j - E(R_j))\right] \\ \text{CoS}(R_i, R_j) &= E\left[(R_i - E(R_i))(R_j - E(R_j))^2\right] \\ \text{CoK}(R_i, R_j) &= E\left[(R_i - E(R_i))(R_j - E(R_j))^3\right]\end{aligned}$$

*Quantitative estimates of marginal contribution of an asset to portfolio risk (volatility and kurtosis in particular)*

# Advanced Techniques for Selection

## *Portfolio Variance, Skewness and Kurtosis*

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- Decomposition of portfolio risk

$$\mu^{(2)}(R_P) = \sum_{i=1}^n \omega_i^2 \mu^{(2)}(R_i) + 2 \sum_{i<j} \omega_i \omega_j \mathbf{CoV}(R_i, R_j)$$

$$\mu^{(4)}(R_P) = \sum_{i=1}^n \omega_i^4 \mu^{(4)}(R_i) + 4 \sum_{i<j} \omega_i \omega_j^3 \mathbf{CoK}(R_i, R_j) + 4 \sum_{i<j} \omega_i^3 \omega_j \mathbf{CoK}(R_j, R_i) + 6 \sum_{i<j} \omega_i^2 \omega_j^2 \mathbf{E}(R_i^2 R_j^2)$$

- Contribution to portfolio risk
  - Start with an initial portfolio: P
  - Compose a new portfolio by adding some amount  $\varepsilon$  of a new asset class or investment style:  $P' = (1-\varepsilon)P + \varepsilon A$
  - Question: under which conditions would P' look better than P?
    - In terms of variance?
    - In terms of kurtosis?

# Advanced Techniques for Selection

## *Contribution to Portfolio Variance*

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- We have that

$$\text{Var}(R_{P'}) - \text{Var}(R_P) \underset{\varepsilon \rightarrow 0}{\approx} -2\varepsilon \text{Var}(R_P) + 2\varepsilon \text{CoV}(R_A, R_P)$$

$$\text{Var}(R_{P'}) \leq \text{Var}(R_P) \Leftrightarrow \beta_{A/P}^{(2)} = \frac{\text{CoV}(R_A, R_P)}{\text{Var}(R_P)} \leq 1$$

- Conclusions (\*):
  - Second moment diversification benefits (decrease in volatility) arise when the additional asset has a (second moment) beta less than or equal to 1 with respect to existing portfolio.
  - Proper normalization of covariance consists of taking a beta equivalent (as opposed to correlation coefficient equivalent).

\* these approximations result from a first order approximation; they are only valid for small  $\varepsilon$

# Advanced Techniques for Selection

## *Contribution to Portfolio Kurtosis*

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- We have that

$$\mu^{(4)}(R_{P'}) - \mu^{(4)}(R_P) \underset{\varepsilon \rightarrow 0}{\approx} -4\varepsilon\mu^{(4)}(R_P) + 4\varepsilon\text{CoK}(R_A, R_P)$$

$$\mu^{(4)}(R_{P'}) \leq \mu^{(4)}(R_P) \Leftrightarrow \beta_{A/P}^{(4)} \equiv \frac{\text{CoK}(R_A, R_P)}{\mu^{(4)}(R_P)} \leq 1$$

- Conclusions (\*):
  - Fourth moment diversification benefits (decrease in kurtosis) arise when the additional asset has a (fourth moment) beta less than or equal to 1 with respect to the existing portfolio.

\* these approximations result from a first order approximation; they are only valid for small  $\varepsilon$

# Advanced Techniques for Selection

## *Application to Traditional Investment Styles*

Higher Moment Betas – from January 1997 through December 2004

### *with S&P 500*

	S&P 500 Growth	S&P 500 Value	S&P 600 Small Cap	US Treasury	US Investment Grade	US High Yield CAA
2nd Moment Beta	1.06	0.94	0.85	-0.07	0.00	0.31
4th Moment Beta	1.00	1.00	0.95	-0.11	0.00	0.41

### *with US Aggregate Bonds*

	S&P 500 Growth	S&P 500 Value	S&P 600 Small Cap	US Treasury	US Investment Grade	US High Yield CAA
2nd Moment Beta	-0.43	-0.64	-0.63	1.22	1.26	-0.20
4th Moment Beta	0.05	-0.11	-0.23	1.22	1.32	0.13

<i>Summary Statistics</i>	<i>S&amp;P 500</i>	<i>US Agg. Bonds</i>
Annualized mean	9.51%	0.29%
Annualized std dev.	16.85%	3.76%
VaR (95%)	7.90%	1.97%
Sharpe ratio*	0.45	-0.45
Skewness	-0.49	-0.81
Kurtosis	2.97	4.29

\*Risk free rate assumed at 2%

# Advanced Techniques for Selection

## *Optimal Mixing with Stocks and Bonds*

*With MSCI World Stock Index*

	Convertible Arbitrage	CTA Global	Event Driven	Equity Mkt Neutral	Long Short Equity
Covariance Beta	0.06	-0.11	0.27	0.06	0.38
Cokurtosis Beta	0.10	-0.26	0.36	0.07	0.38

*With Lehman Global Treasury Bond Index*

	Convertible Arbitrage	CTA Global	Event Driven	Equity Mkt Neutral	Long Short Equity
Covariance Beta	-0.06	1.51	-0.34	0.05	-0.37
Cokurtosis Beta	-0.12	1.27	-0.36	0.08	-0.08

Co-moments of Hedge Fund Index Return Distribution with Respect to Stock and Bond Returns,  
based on Edhec Hedge Fund Indices with Respect to MSCI World Equity and Lehman Global Treasury Bond Index over the Period  
01/1997-12/2005

	No diversification potential
	Low diversification potential
	High diversification potential
	Very high diversification potential

### Interpretations (improvement of portfolio moments)

Decrease in portfolio 2<sup>nd</sup> moment (volatility)  $\Leftrightarrow$  beta covariance  $< 1$

Decrease in portfolio 4<sup>th</sup> moment  $\Leftrightarrow$  beta cokurtosis  $< 1$

# Advanced Techniques for Selection

## *Selection of Hedge Fund Strategies*

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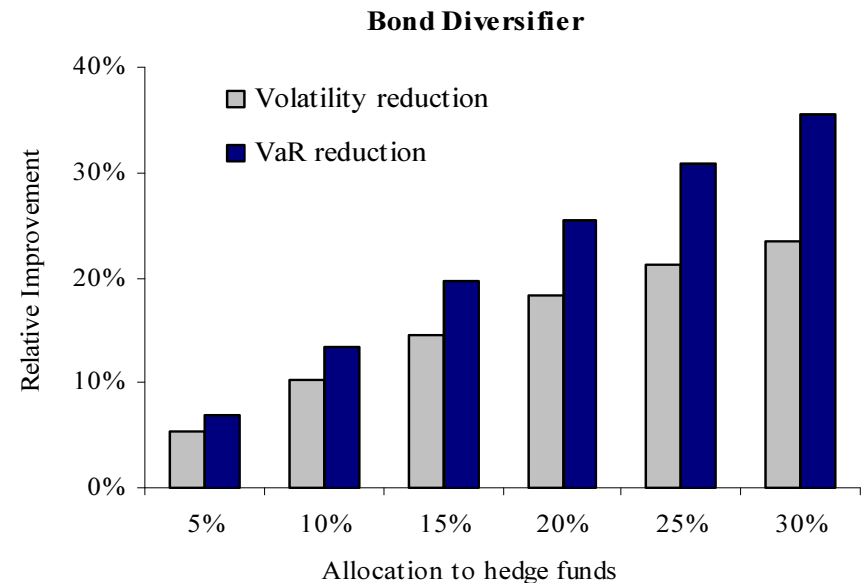
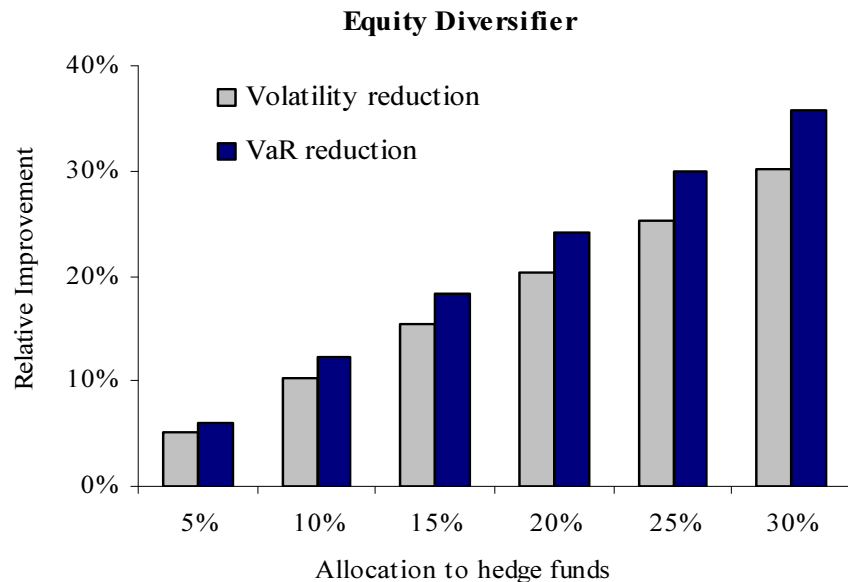
- Selecting the right strategies for diversifying either a stock or bond portfolio.
- For example, we do not expect long-short equity, which contains lots of TEF, to help diversify a stock portfolio.
- Based on the above analysis, we have found that the following mix is well suited for the purpose of diversifying a stock versus a bond portfolio.

<b>Equity Diversifier</b>	<b>Bond Diversifier</b>
Convertible Arbitrage	Convertible Arbitrage
CTA Global	Equity Market Neutral
Equity Market Neutral	Event Driven
	Long/Short Equity

# Advanced Techniques for Selection

## *Risk Reduction Benefits are Sizable*

- The suitably designed diversifiers allows for a significant reduction in risk (both average and extreme risk), even for relatively modest allocations to hedge funds.



The sample period is 04/2002 to 12/2005. For diversifying the bond portfolio, we have used the following Edhec Investable Hedge Fund Indices: Equity Market Neutral, Convertible Arbitrage, Event Driven, Long/Short Equity. For diversifying the equity portfolio, we have used the following Edhec Investable Hedge Fund Indices: Equity Market Neutral, Convertible Arbitrage, CTA Global. The MSCI World Equity Index and the Lehman Global Treasury Index have been used as proxies for the returns on stocks and bonds, respectively.

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# Enhanced Techniques for Parameter Estimation

## *Challenges in Optimal Beta Management*

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- State-of-the art-techniques for second moment estimates
  - Constant correlation approach (Elton and Gruber (1973))
  - Single factor forecast (Sharpe (1963))
  - Multi factor forecast (e.g., Chan, Karceski and Lakonishok (1999))
  - Optimal shrinkage towards the constant correlation (Ledoit and Wolf (2004)) or towards the single-factor model (Ledoit (1999))
  - Portfolio constraints (Jagannathan and Ma (2000))
  - Conditional (GARCH) covariance matrix estimates
- State-of-the art-techniques for first moment estimates
  - Single factor forecast (Sharpe (1963))
  - Multi factor forecast (e.g., Chan, Karceski and Lakonishok (1999))
  - Black & Litterman (1992)
- Extending these techniques to hedge fund portfolios
  - Enhanced estimates of co-skewness and co-kurtosis parameters (Martellini and Ziemann (2005))
  - Extending B&L beyond the mean-variance setup (Martellini, Vaissié and Ziemann (2006))

# Enhanced Techniques for Parameter Estimation

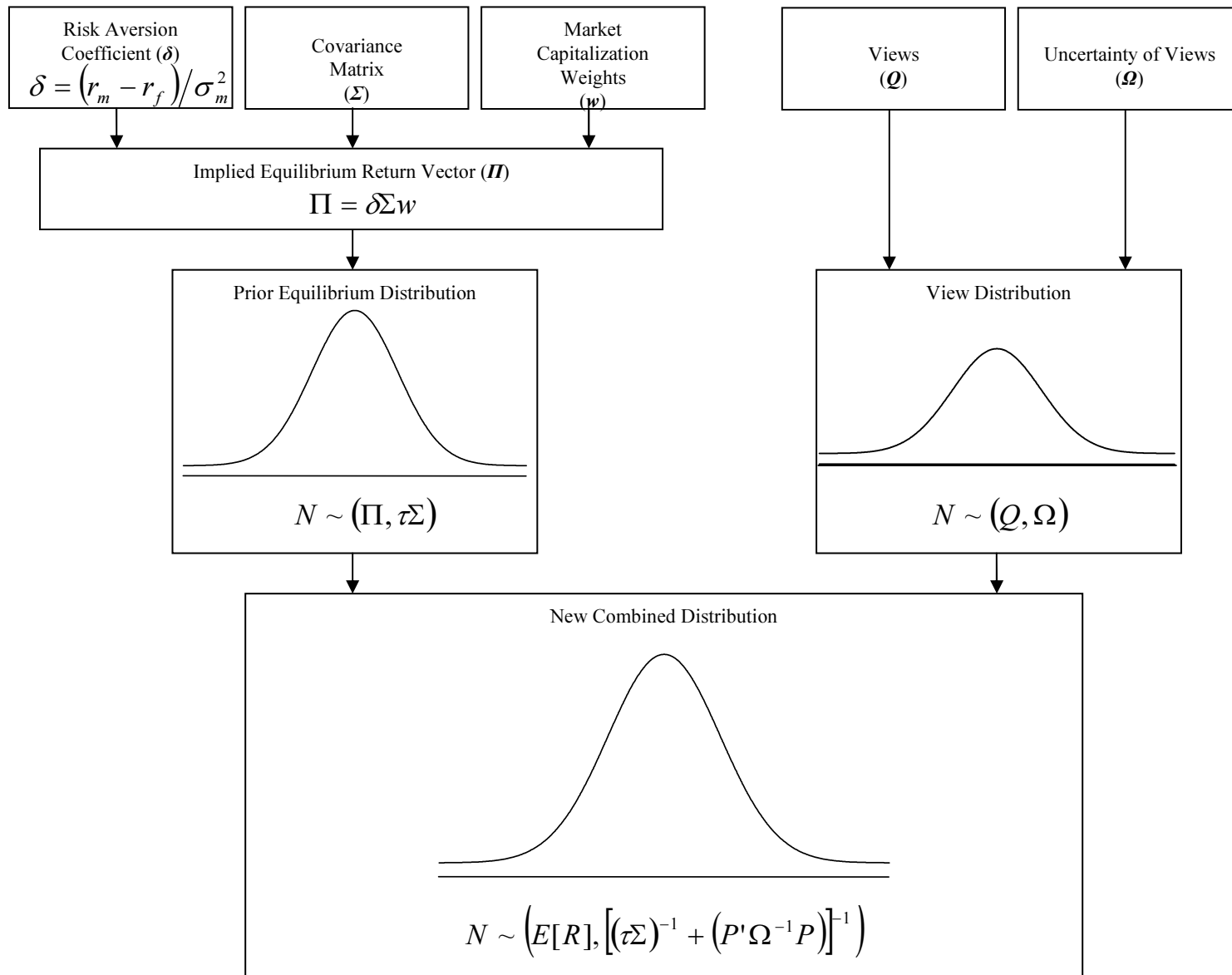
## *Enhanced Techniques for Expected Return Estimation*

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- Problems with optimization: optimal portfolio weights
  - Are very sensitive to changes in expected returns
  - Tend to be hard to interpret (often inconsistent with *naive* diversification)
- Problems with estimation
  - Merton (1980), Jorion (1985): optimal estimator of the expected return is noisy with a finite sample size
  - The estimator of the variance converges to the true value
- Sophisticated portfolio construction methods can be used by investors to overcome the problem of unintuitive, highly-concentrated, input-sensitive portfolios.
- Black-Litterman asset allocation model uses a Bayesian approach to combine the subjective views of an investor regarding the expected returns of one or more assets with market equilibrium vector of expected returns (the prior distribution) to form a new, mixed estimate of expected returns.

# Enhanced Techniques for Parameter Estimation

## *Black-Litterman Approach*



# Enhanced Techniques for Parameter Estimation

## *Implied Neutral Views*

$$\Pi = \lambda \Sigma w_{mkt}$$

- $\Pi$  is the excess market returns over the risk free rate
- To reverse engineer this portfolio, we need
  - $\lambda$  (Lambda) is the risk aversion coefficient
  - $\Sigma$  (Sigma) is the covariance matrix of returns
  - $w_{mkt}$  is the market capitalization weight of the assets

$$\begin{pmatrix} E(R_1) - r_f \\ \vdots \\ E(R_n) - r_f \end{pmatrix} = \frac{E(R_M) - r_f}{\sigma_M^2} \begin{pmatrix} \text{cov}(R_1, R_M) \\ \vdots \\ \text{cov}(R_n, R_M) \end{pmatrix}$$

$\text{cov}(R_1, R_M) = \text{cov}\left(R_1, \sum_{i=1}^n w_i R_i\right) = \sum_{i=1}^n w_i \text{cov}(R_1, R_i)$

This is the standard CAPM!

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# Extending Black-Litterman Analysis

## *Allocation to HFs with Active Views on Expected Returns*

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- The benefits of active style rotation can be added to the benefits of diversification to ensure the design of an even better investment solution.
- In a recent paper (Martellini, Vaissié, Ziemann, forthcoming in the JPM), we have extended Black-Litterman analysis to account for the presence of non-trivial preferences over higher order moments of hedge fund return distribution.

- Use 4-moment CAPM for extracting neutral views:

$$\mu - R_0 = \alpha_1 \beta^{(2)} + \alpha_2 \beta^{(3)} + \alpha_3 \beta^{(4)}$$

- One may use as a benchmark any target portfolio from which we obtain neutral expected return estimates (not necessarily a market cap weighted portfolio).

# Extending Black-Litterman Analysis

## *Generating Active Views on Expected Returns*

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- Next step consists of generating active views on hedge fund style returns.
- There is strong evidence of predictability in hedge fund returns (Amenc, El Bied and Martellini, FAJ, 2002).
- In illustration below, active views come from an (in-sample) conditional 3-month lagged factor analysis of hedge fund returns.
- The predictive factors we use are:
  - Implied Volatility (VIX) - CBOE SPX Volatility VIX
  - First differences of the implied volatility
  - Commodity Index - Goldman Sachs
  - Term Spread - Lehman US Treasury 5-7 years minus Lehman US Treasury 1-3 years
  - Credit Spread - Lehman US Universal: High Yield Corp. Red Yield minus Lehman US Treasury 1-3 years
  - Value vs. Growth - S&P 500 Barra/Value minus S&P 500 Barra/Growth
  - Small Cap vs. Large Cap - S&P 600 Small Cap minus S&P 500 Composite
  - S&P 500 Composite return
  - T-Bill - Merrill Lynch T-Bill 3 month
  - US Dollar - US MAJOR CURRENCY MAR73=100 (FED) EXCHANGE INDEX
  - Bond return volatility - calculated over one month Lehman US Aggregate return

# Extending Black-Litterman Analysis

## *Illustration*

- We have considered three different active Black-Litterman portfolios associated with different tracking error levels, based on the different values for a parameter  $\tau$  that allows us to fix the respective contribution of active versus neutral views in the B&L approach.
- The active style selection process, combined with the Black-Litterman portfolio selection method, allows for significant out-performance without a large increase in tracking error, as can be seen from the information ratio values. The excess performance, as well as the tracking error, increase in  $\tau$ , as expected.

	PF minVaR	PF Black Litterman $\tau = 1$	PF Black Litterman $\tau = 5$	PF Black Litterman $\tau = 20$
Mean annual return	8.79%	9.79%	10.48%	10.71%
Volatility	4.29%	4.32%	4.39%	4.46%
VaR (95%)	1.33%	1.21%	1.17%	1.19%
Sharpe ratio (r=3%)	1.58	1.80	1.93	1.95
Tracking error		0.86%	1.24%	1.37%
Information ratio		1.17	1.37	1.41

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# Conclusions

## *From a Sell-Side to a Buy-Side Perspective*

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- Most current hedge fund offerings to institutional investors are based on the promotion of hedge funds as attractive stand-alone absolute return vehicles.
- This is a serious problem because
  - Hedge funds are not a homogeneous asset class and different hedge fund strategies are exposed to different risk factors.
  - It accounts neither for the investor's preferences and constraints nor for their existing portfolios.
- The times are changing
  - Recognising that individual strategies matter allows for proper allocation decisions.
  - Better performance and risk management can be generated from such strategic asset allocation decisions.
- More meaningful hedge fund solutions can be designed, based on the recognition that contrasted factor exposures of various hedge fund strategies allow for better management of investors' core portfolios.